A MODEL FOR PRE-SERVICE SERVICE TEACHING EDUCATION FOCUSED ON THE DEVELOPMENT OF MATHEMATICAL KNOWLEDGE FOR TEACHING: PERCEPTION OF FUTURE TEACHERS

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This paper reports the main results of a study on a didactic device that seeks to support mathematics courses of pre-service education programs for Primary School teachers. This device was designed to support the development of Mathematical Knowledge for Teaching, the collaborative construction of mathematical knowledge and situated learning. The preliminary results, based on the analysis of the perception that future teachers have of the device, suggest that it reflects in a coherent manner the principles that guide its design.

INTRODUCTION

In recent years, there has been much concern about the quality of pre-service teacher education (PTE), particularly in relation to the lack of opportunities to develop essential competencies to effectively lead teaching-learning processes in mathematics. According to Ball, Thames and Phelps (2008), a key aspect of this proficiency is the specialized knowledge that the authors conceptualize in their Mathematical Knowledge for Teaching (MKT) model. Special emphasis has been placed in the role that mathematics teacher educators (MTE) have in providing the learning opportunities to develop the MKT, because they are the ones who make this knowledge accessible and can support the process of linking theory with practice. Moreover, through their lessons they impact the future teaching practices of students (Boyd, 2014).

Various approaches have been proposed to enhance the MKT of pre-service teachers (PTs) such as establishing instruction focused on problem solving, involving them in activities that emphasize the communication of mathematical ideas, and the development of fundamental teaching practices (Ball & Forzani, 2011).

Chile also faces the challenge of improving PTE. The TEDS-M international study, which compares the performance of PTs from 17 countries, placed Chile second to last in disciplinary and pedagogical knowledge of mathematics, and below countries with a similar or even smaller per capita income (Tatto *et al*., 2012). This can be observed

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in the fact that most PTs have problems working with basic operations, and difficulties relating different mathematical concepts and developing arguments.

Other studies highlight the deficit of the PTE system. According to Ávalos (2014), PTs study in a system with great diversity, which often does not meet minimum quality standards. Moreover, a study focused on characterizing primary school PTE showed that many MTE have precarious working conditions and do not have opportunities for professional development (Mineduc, 2016). The same study showed that the math courses in these programs do not meet the requirements of the new school curriculum, particularly regarding the development of mathematical skills.

In view of this need, the Laboratory of Education of the Center for Mathematical Modelling (CMM) of the Universidad de Chile is developing a R+D project that has two main goals: first, to increase MKT in PTs through activities that foster inquiry, the analysis of learning situations, and the development of mathematical skills; second, to support and guide MTEs' use of active learning methodologies. In the project, Learning Units for teacher training are being designed, which are sequences of lessons around a mathematical topic of high impact for PTE. The units include mathematical tasks for teaching and supporting resources for MTEs.

This study focuses on the pilot experiences of the Learning Units. We sustain that these units have a significant impact in the perceptions of the mathematical teaching of PTs participating, and that those perceptions acknowledge some of the principles that guided the design of the Learning Units. Considering this as an ongoing project, we address the following research question: How do PTs participating in the pilot recognize and value some of the principles that guided the model's design?

The article is structured as follows: The first part provides a description and justification of the Learning Units design, including an overview of the three principles analyzed in this work. This is followed by an explanation of the research methodology. Finally, there is an analysis of the study's results and its main conclusions.

DESCRIPTION OF THE LEARNING UNITS

The Learning Units were developed by a multidisciplinary team composed of mathematics teachers, mathematicians and MTEs from several universities across the country. The development followed an elaboration-testing-adjusting design cycle. Four units were developed during 2017, whose topics were selected for their high impact on initial training and the feasibility of being tested by the MTEs. Two of the units deal with Numbers: *Addition and Subtraction Problems* (N1), that covers the classification of these problems according to the actions involved and the place of the unknown (Lewin, López, Martínez, Rojas & Zannoco, 2010); and *Representing addition and subtraction problems* (N2), which seeks to identify concrete and pictorial representations of these problems and discussing their pertinence (Veloo & Parmijt, 2017). In addition, two Geometry units were developed: *Definition of perimeter* (G1), which addresses the process of constructing a definition of a contour of a shape, and problem solving involving perimeters (Lu, Weng & Tuo, 2013); and *Variations of area* *and perimeter* (G2), which deals with the relationship between area and perimeter when changing geometric shapes (Ma, 1999).

Each unit is designed to be used in two consecutive 90-minute lessons, and includes a sequence of 4 or 5 activities. The supporting material for the students consists of worksheets and a lesson plan for the MTE, which includes the purpose, instruction modality, possible student answers, teaching suggestions, pedagogical notes of each activity, and recommendations to carry out the transitions between activities.

Principles for the design of the Learning Units

The Learning Units developed through this project correspond to a didactic device whose design considers a series of principles, such as:

- To offer opportunities to develop *Mathematical Knowledge for Teaching* (MKT) (Ball, 2008).
- To foster a *collaborative construction of mathematical knowledge* (Ball & Bass, 2000), which acknowledges the role of interactions between students mediated by the MTE.
- To promote a teaching process that recognizes *situated learning* (Brown, Collins & Duguid, 1989) as a way to favor learning to teach through activities that bring PTs closer to specific tasks in their future professional work.

According to the MKT model, the Learning Units focus mainly on Specialized Content Knowledge (SCK), Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT). In the Geometry units, the emphasis was on the SCK. The proposed activities seek that PTs construct a definition of the boundary and perimeter of a plane shape, solve problems related to perimeters using visual, inductive and deductive reasoning, and develop arguments to analyze statements about changes of area and perimeter. On the other hand, the Numbers units mainly address KCS and KCT. In them we find activities whose purpose is for PTs to observe different actions and recognize the various types of addition and subtraction problems, distinguish those that are easier or more difficult to solve for a child, identify concrete and pictorial representations for these problems and what motivates the transit from one to another.

With respect to the second principle, the methodology of Mathematical Discussion (Chapin, O'Connor and Anderson, 2003) was adopted as a strategy to encourage classroom interactions that can lead to the collaborative production of mathematical knowledge. Thus, the lesson plan includes whole-class discussions in most of the activities and provides teaching suggestions with questions to foster the discussion.

The situated learning approach was incorporated by using classroom contexts, such as activities based on the analysis of videos, student productions and case studies. These activities seek to bring the PTs closer to didactical problems inside school classrooms. An example is the use of video clips of a child solving different types of addition and subtraction problems to motivate reflection on the difficulty in their resolution.

METHODOLOGY

This study builds on pilots for the Learning Units applied in the third week of classes in mathematics courses for primary-school PTE programs of two universities in Santiago (A and B). Prior to the implementation, the MTEs were able to review in detail the planning of the units, and agreed to adjust to the guidelines and suggested times. During the implementation, the students were aware that all the activities carried out, as well as the material used in classes, were done in the context of a project.

A qualitative research approach was used to understand the perspective of students regarding their experience with the applied Learning Units (Flick, 2002). Focus groups, applied immediately after the end of the implementation, were used as a data collection technique, as they facilitated dialogue and discussion among participants, contributing to the exchange of ideas, opinions and reflections (Kidd & Parshall, 2000). A sample of 35 students volunteered to participate, distributed as the following table shows:

Course	University	Learning Units	Participants
1	A	N1 and N2	6
2	A	G1 and G2	7
3	B	G1 and G2	9
4	B	G1 and G2	4
5	B	G1 and G2	9

Table 1: Summary of the sample of the study.

Each focus group was guided by two researchers, a math teacher and a researcher in education. They had an average duration of one hour and included a set of guided questions aimed at understanding the perceived main learnings achieved, the types of mathematical knowledge that were developed and the teaching practices adopted by the MTE. All the focus groups were recorded and subsequently transcribed.

The preliminary analysis was carried out by two members of the research team, who coded the responses in several emerging categories. These categories were defined through a Constant Comparative Method (Strauss & Corbin, 2007). Those segments were then cross-referenced with the three principles considered in this study and reviewed once again. To ensure the reliability of the results, they were also cross-checked by the members of the research team.

RESULTS

We will now examine the main results obtained in the analysis, with quotes that were selected to faithfully convey the meaning intended in the evidence. Each quote is identified by the course number described in Table 1.

The first interesting result is that in all the focus groups, the PTs recognize that the lessons were different from the ones they were used to, considering their school and university experiences, as it is seen in the following quotes:

> Something I want to say about the class is that in school, at least in my case, I was used to being given the formula, and I only applied it. But here, it is how I discover things (FG Course 2).

> I noticed a lot the difference when these four classes started. It was very clear the difference with respect to how we had been working previously (FG Course 4).

> What was also very noticeable was the participation that each one had. Because in previous classes we write and [only] the one who knows the content speaks. However, in these classes it was not like that, because one could say what one thought about the content and see if it really was that way or not. So that is what Peter said about constructing our own learning (FG Course 4).

As these quotes suggest, the PTs recognize the change in teaching methodology that the device promotes, which involves a more active participation of students.

The results that follow are organized according to the three principles that underlie the Learning Unites design:

Development of Mathematical Knowledge for Teaching

The students pointed out that the Learning Units contributed positively to the development of different types of mathematical knowledge. Faced with the question "What have you learned throughout these lessons?", the PTs recognize that the units develop knowledge they consider relevant for their future teacher practice. In addition, their description suggest that they identify certain learnings related to the SCK, KCT and KCS, which coincide with the types of knowledge that promote the activities, as the following quotes show:

> Because we have the idea of a perimeter, which apparently, we all thought was right, [...] and suddenly someone comes out and says no, [...], and you realize that you must agree. And it seems that an idea as basic as a perimeter, something that perhaps we should have very clear, is not so clear. And that [it is] interesting to agree to remake a definition. (FG Course 2, SCK).

> When we were in school it was not like this, they give the problem and it was more intuitive, "ah, I only have to add" and ready, but I did not know "this is a comparison or transformation [problem]". So, in that sense, as Denisse said, being able to classify helps you as a teacher in the future to formulate problems and that was, for me, the most important thing we saw in these four classes. (FG Course 1, KCT).

> Sure, analyze or deduce what the child might do. I think that was a very important discussion topic: how to think about what a child would do, and not about what I would do. (FG Course 1, KCS).

Collaborative construction of mathematical knowledge

Another interesting result is that the PTs recognized that learning was built collaboratively through the sequence of activities, highlighting Mathematical Discussion as an element that significantly contributes to this process. The PTs distinguished various ways in which it can be articulated. First, they recognized the role of the MTE as a mediator, as can be seen in the following quote:

> It was very interesting that when someone had a question and asked it, the teacher expected us to answer it ourselves, to try to explain it. And it was very cool, because it was very hard to explain. (FG Course 2).

In other cases, the Mathematical Discussion was perceived as a means for building knowledge among peers. This was positively valued, although sometimes it caused concern about the conclusions reached through these discussions:

> S1: At one point, for example, if the majority said that it was such and such thing and another smaller group said that it was not, that it was the other one, then they had to defend why they thought it was so. Then, that generated arguments. That was, like, I do not know, it was cool. But on the other hand it was also confusing, because all the arguments were good.
>
> S2: And there was a problem because we came to the supposed conclusion while the ideas were still up in the air and immediately another exercise sheet came, so [the conclusion] was like getting lost.
>
> S1: That debate was cool, but if it had been left more settled, it would have been better (FG Course 1).

It is relevant to observe how the interactions proposed in the device, which sought to promote that the MTE involves the work of the groups and leads whole-class discussions, was highlighted by the PTs.

Situated learning

One way in which the students identified that the units brought the school classroom closer to the PTE is the concrete use of classroom contexts in the activities:

> When they showed the videos like this: the child would end up adding and they would ask him to subtract and it was like, "Oh, he was wrong for this reason. And then you had to analyze and think about how the child had made the mistake. (FG Course 1).

On the other hand, the data shows that students also approach the classroom symbolically, that is, when they reflect on their learning experiences in relation to the challenges they will face in their future teaching work:

> What he is looking for is our knowledge about the topic, and then he takes our knowledge, which is going to be very diverse, and that is what happens in the classroom: one exposes a case in the classroom and all the children have different thoughts about that topic, and that is what the activities are looking for somehow. [...]

> So, it is not so much about the discipline, but how we are going to, somehow, model how children will learn. (FG Course 3).

The interesting thing about this is that it shows how students begin to position themselves as future teachers because of their learning experience:

> That I think [...] that we are just learning how to stop thinking of ourselves as students and [begin to] think of ourselves as teachers. Because before, when we are students, we are kind of focused on our way of developing problems. But [now with these activities] we have to put ourselves in the child's place and see the different approaches that children can use and be prepared to see the big picture. (FG Course 1).

DISCUSSION AND FUTURE WORK

The aspects highlighted by the PTs suggest that the Learning Units support lessons consistent with the three principles that underlie their design, thus contributing to developments in the PTs mathematical and didactic knowledge relevant to their future work. This would confirm the feasibility of using these types of devices to support PTE in mathematics.

The evidence suggests that Mathematical Discussion contributes to making PTs aware of the idea that learning has a significant social component and that mathematical knowledge can be constructed collaboratively (Putnam & Borko, 1997). Although the PTs valued the discussions as instances to share their ideas and thinking, they made evident at times the lack of systematization at the close of the discussions.

On the other hand, it is observed that learning becomes more significant for the PTs when they are placed within a classroom context. It is striking that the reflection concerning the classroom not only takes place in the activities designed for that purpose, but also in the way they connect their learning experience with those of their future students when addressing other mathematical tasks. A possible explanation for this is that the activities that entail classroom situations trigger a permanent reflection regarding the teaching endeavor. This would also be a contributing factor to their perception as future teachers.

Finally, the study shows the need for an in-depth analysis of the data to answer questions such as:

i) What improvements to the design of the device could help the discussions to better systematize closure ideas?

ii) What characteristics of the didactic device trigger pedagogical reflection in PTs and contribute to making them envision themselves as future teachers?

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