

Being a Girl Mathematician: Diversity of Positive Mathematical Identities in a Secondary Classroom

Author(s): Darinka Radovic, Laura Black, Christian E. Salas and Julian Williams

Source: *Journal for Research in Mathematics Education*, Vol. 48, No. 4 (July 2017), pp. 434-464

Published by: National Council of Teachers of Mathematics

Stable URL: <http://www.jstor.org/stable/10.5951/jresemetheduc.48.4.0434>

Accessed: 07-07-2017 10:21 UTC

REFERENCES

Linked references are available on JSTOR for this article:

http://www.jstor.org/stable/10.5951/jresemetheduc.48.4.0434?seq=1&cid=pdf-reference#references_tab_contents

You may need to log in to JSTOR to access the linked references.

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at

<http://about.jstor.org/terms>



National Council of Teachers of Mathematics is collaborating with JSTOR to digitize, preserve and extend access to *Journal for Research in Mathematics Education*

Being a Girl Mathematician: Diversity of Positive Mathematical Identities in a Secondary Classroom

Darinka Radovic

Universidad de Chile, Chile

Laura Black

The University of Manchester, United Kingdom

Christian E. Salas

Universidad Diego Portales, Chile

Julian Williams

The University of Manchester, United Kingdom

The construction of positive mathematical identities (MIs) is a complex and central issue in school mathematics, where girls are usually “counted out” of the field. This study explores positive MIs (high achiever and positive relationship with mathematics) of 3 girls. We employed a nested model of identity based on a case study approach (i.e., female mathematics students nested within a cluster of students nested within a mathematics classroom). The results highlight diversity in how these girls experienced mathematics: They valued different forms of doing mathematics (independent and collaborative; wider and complex; and straightforward, procedure-oriented mathematics), showed different forms of engagement (detachment, protagonist and challenging, and compliant and support-seeker), and narrated different MIs (efficient, different, and responsible). The study also explores and discusses the roles of mathematical practice and belonging to different peer clusters in these different forms of identification.

Keywords: Classroom interactions; Gender; Mathematical identities; Narratives; Peer relationships; Positive identities

Several studies over the last few decades have documented how women tend to live conflicted mathematical lives, dealing with relative underperformance as a group and the social discourses documenting these differences, as well as participating less than men in postcompulsory mathematics and related careers

This work was funded by the National Commission for Scientific and Technological Research in Chile (CONICYT) and by the Center for Mathematical Modeling through a Basal project (CMM) and Unité Mixte Internationale UMI CNRS 2807. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the CONICYT or the Center for Mathematical Modeling. We are also grateful for Man-Chin Leung’s assistance with the software, SCM-4.

(e.g., Lubienski & Ganley, in press; Organisation for Economic Co-operation and Development [OECD], 2015). When attempting to explain these difficulties, studies have predominantly taken a psychological perspective, trying to identify what is “wrong” with girls or women in their relationships with mathematics—an approach that has been criticized as “blaming the victim” (Anyon, 1981; as cited in Boaler, 1997, p. 284). A key critique of such work suggests that by focusing on girls’ individual psychological characteristics as inherent fixed traits, a naturalization process may take place wherein girls are seen as ill-suited to mathematics (Boaler, 2002b). In addition, this psychological approach neglects the fact that the process of learning mathematics does not occur inside an isolated individual in a historical vacuum but rather is embedded in a wider and a specific social context (Boaler, 2000; Kaiser & Rogers, 1995). Consequently, some have suggested that, in order to understand students’ relationships with mathematics (and with any other social activity), the emphasis needs to be placed on the relationship between the individual and the social context (e.g., Holland, Lachicotte, Skinner, & Cain, 1998). It is here where the concept of identity has been suggested as a useful lens with which to explore the relationship between students and mathematics (Langer-Osuna & Esmonde, in press; Sfard & Prusak, 2005), with particular power for understanding the experiences of underrepresented and underperforming groups (e.g., Martin, 2012; Varelas, Martin, & Kane, 2012).

Recently, a growing number of studies in mathematics education have used concepts such as identity or *mathematical identity* (MI) from a social perspective (Darragh, 2016; Stentoft & Valero, 2009). In studies investigating girls’ or women’s relationships with mathematics, a strong contrast can be found between studies in compulsory education and studies in postcompulsory education. This is also the case with more general research on MIs (see Radovic, 2015). Research on postcompulsory mathematics has mainly explored women’s MIs by capturing the singularity of their experiences and their reflections on those experiences, using a narrative approach (Herzig, 2004; Mendick, 2005a, 2005b; Rodd & Bartholomew, 2006; Solomon, 2012; Solomon, Radovic, & Black, 2015). It has been noted that women with a positive identification with mathematics, those who have decided to pursue mathematics as a career path, must negotiate a series of complexities and conflicts in order to engage with mathematics. Some of the complexities described in the literature include the role of others in building a mathematical story (Solomon, 2012), the power of discourses that define mathematics as a male domain (Mendick, 2005a), and the positioning of women as an invisible minority in a male peer context (Rodd & Bartholomew, 2006). A main proposition of this line of research is that women need to perform complex “identity work” when engaging with mathematics (Black & Williams, 2013; Holmegaard, Madsen, & Ulriksen, 2014; Mendick, 2005a; Solomon, 2012), implying that becoming a female mathematician is not easy.

In contrast with studies on postcompulsory education that focus on a positive (albeit complex) identification with mathematics, studies in compulsory education have tended to focus on how girls interact with mathematical activity in schools,

paying particular attention to how they disengage from it. This line of research has suggested that traditional mathematics teaching practices—in which teachers have a central role, individual and competitive work is dominant, and rote repetition and rule-bound learning are encouraged—contribute to girls' difficulties and disengagement from mathematics (Atweh & Cooper, 1995; Black, 2004; Boaler, 2002a; Boaler & Greeno, 2000; Solomon, 2007). Although both boys and girls have been observed as disengaging from these traditional practices (Nardi & Steward, 2003), boys have been shown to be better able to reposition their goals by pursuing competition and relative success without disengaging. Girls, in contrast, have been observed to find this repositioning more difficult, thus tending toward disengagement and disidentification from mathematics (Boaler, 2002a, 2002b). It is interesting to note that some of these studies have anecdotally reported the existence of girls with positive relationships with mathematics who engage in mathematical activity even in traditional mathematics classrooms (e.g., Solomon, 2007). However, accounts of how such “against the odds” positioning is made possible within traditional school mathematical practices and how it is experienced by such girls constitute an underresearched area.

The mediation of peer relations is an area of investigation that could develop our understanding of how girls' positive MIs are made possible in the classroom. In engaging in the activity of building intimate relationships with peers, adolescents form their personal meaning of life through social consciousness diverted inward (El'konin, 1971). Adult norms reproduced in peer relations become relevant identity resources because engaging in peer relations is the leading activity at this stage of development (Karpov, 2003; Leontyev, 2009). Peer relations in the classroom have been regarded as a source of social comparison relevant in the development of adolescents' self-concept and self-efficacy (e.g., Marsh, 1987; Marsh, Trautwein, Lüdtke, & Köller, 2008; Zeidner & Schleyer, 1999) and as influential on the values and meanings that students give to education and engagement (Francis, Skelton, & Read, 2010; Jackson, 2006).

In mathematics, recent studies have explored how belonging to different peer groups in the classroom can play a role in the process of MI construction. These studies have mainly considered how friendships and peer relationships are forms of capital (Choudry, Williams, & Black, 2016) that can help students negotiate access to mathematical resources (Gholson & Martin, 2014) or that students can use to negotiate more powerful positions when they perceive themselves as less powerful in mathematical practice (i.e., with less mathematical capital; Esmonde & Langer-Osuna, 2013). Although these studies have provided support for the existence of relationships between peer dynamics and the nature of mathematics in the classroom, they have conceptualized peer relationships as a form of capital but not necessarily as a source of identity. In other words, they have not explored how constructions of *who we are* may have an impact on notions of *who I am* in relation to mathematics.

In summary, previous studies have alerted us to the conflicts and contradictions that girls and women face in being or becoming successful in mathematics.

Although studies in postcompulsory education have suggested that these conflicts are related to women's difficulties in finding a narrative of themselves as mathematically successful, studies in compulsory education have not consistently researched this issue. In addition, we hypothesize that these identity negotiations are related to girls' peer relationships. Although engaging in these relationships is the dominant activity in adolescence, we found no research on how girls' peer relationships influence classroom dynamics or how they might mediate the formation of positive MIs. Therefore, this article systematically reports how a group of mathematically successful girls perform positive MIs in compulsory education during early adolescence (13–14 years old). In particular, this study explored how these girls' relationships with their peers (and their peer cluster identity) mediated girls' identification with mathematics by examining how these relationships were used to maintain or negotiate different positions of engagement and success.

Theoretical Model of Identity and Methodological Rationale

We adopted a sociocultural conceptualization of identity following the synthesis by Holland, Lachicotte, Skinner, and Cain (1998) of Vygotskian, Bakhtinian, and Bourdieusian approaches. This synthesis conceptualizes identity as a dynamic and relational process composed of a *contradictory unit of discourses and practices*, a dialectical relationship between narrative stories and forms of participation or acts. "Others" are central to this dialectical relationship. Identities develop in relation to others, using the voices of others, and during a process in which individuals position themselves and are positioned by others as certain *kinds of people*. Others also become available figures or kinds of persons with whom students can identify or disidentify with. In this negotiation process, students often develop a sense of belonging to different peer clusters, which become an important resource for identity construction and become a part of identity itself (Gholson & Martin, 2014; Lim, 2008; Renold & Allan, 2006).

The contradictory unit of discourses and practices represents how identities develop in a dialectical relationship between narrative stories and forms of participation or acts. Narrative stories are mediated by the use of symbols that are learned in practice and thus are context-specific and become internalized through social interaction (Holland & Lachicotte, 2007). Students learn to identify themselves with different figures, cultural models, or types of persons, such as "nerd," "geek," "freak," "capable," or "at risk" (Holland & Skinner, 1987), which then become resources for further reflection in inner speech (Holland & Lachicotte, 2007). In contrast, forms of participation or acts do not necessarily rely on internalization but are related to regularities and inconsistencies in the way people engage with others and are treated by others in practice (Williams, Davis, & Black, 2007). However, both narratives and acts are expressions of the kind of person an individual sees himself or herself as being. As Holland et al. (1998) put it, "People tell others who they are, but even more important, they tell themselves and then try to act as though they are who they say they are" (p. 3). Similarly, one's

engagement with others, one's acts (in joint activity whether in the classroom, at home, or in interviews), and how these acts are seen and positioned by others, also mediate the stories that one tells about who one is (both to oneself and to others). Although some consistency is expected, inconsistencies and conflicts between narratives and acts are also usually found. Thus, identity development is a dialectical process.

Both narratives and acts emerge from a process of self-positioning in relation to two main aspects of mathematical practice. First, based on the principle that an individual is always nested within a peer cluster, girls' narratives and acts can be understood only in relation to the narratives and acts of the peer clusters that they affiliate with and the peer clusters that they distance themselves from (notions of *we are* and *we do* and *they are* and *they do*; Lim, 2008). In this sense, cluster identities are also understood as particular narratives and acts built over time through engagement in activity and through social acknowledgment and recognition. Particularly relevant to cluster identities is the recognition of a (flexible) border or limit that defines who is and who is not part of the cluster—who we are and who the others are (that we are not).

Second, both narratives and acts will always be considered in relation to the wider context of the classroom and the particular social context that will delineate the meaning of mathematics (and doing school mathematics) and gender (and being a girl in school) in this particular cultural context (Atweh & Cooper, 1995). In other words, this study attempts to capture how each of the girls and her peer cluster engage with, comply with, or resist classroom mathematical norms and practices (e.g., Cobb, Gresalfi, & Hodge, 2009; Solomon, 2008); how they relate to the particular participation structures in place (Langer-Osuna, 2011); and how they negotiate different notions of what being a girl or a boy mean in the mathematics classroom (e.g., Mittelberg, Rozner, & Forgasz, 2011).

To account for this complexity, we used a nested model of identity based on a case study approach (Yin, 2003). Such a model proposes that a case (i.e., a female mathematics student) can be nested within another case (i.e., a cluster of students inside a mathematics classroom) that can also be nested within a larger case (i.e., the entire mathematics classroom), all of which is nested in a particular social context. Through the use of a nested model of identity, this study explored the process of identity construction of three girls belonging to three different peer clusters in one mathematics classroom. See Figure 1 for a visualization of this MI model.

Methods

Chilean Context

This study was based in Chile, a country where the relationship between girls and mathematics is still seen as problematic. Girls have been consistently shown as underperforming (Agencia Calidad de la Educación, 2014; Ministerio de Educación Chile, 2005), and studies have documented that teachers and students hold stereotypical expectations about girls' achievement and dispositions in this subject (del Río & Strasser, 2013; Mizala, Martínez, & Martínez, 2015). Although

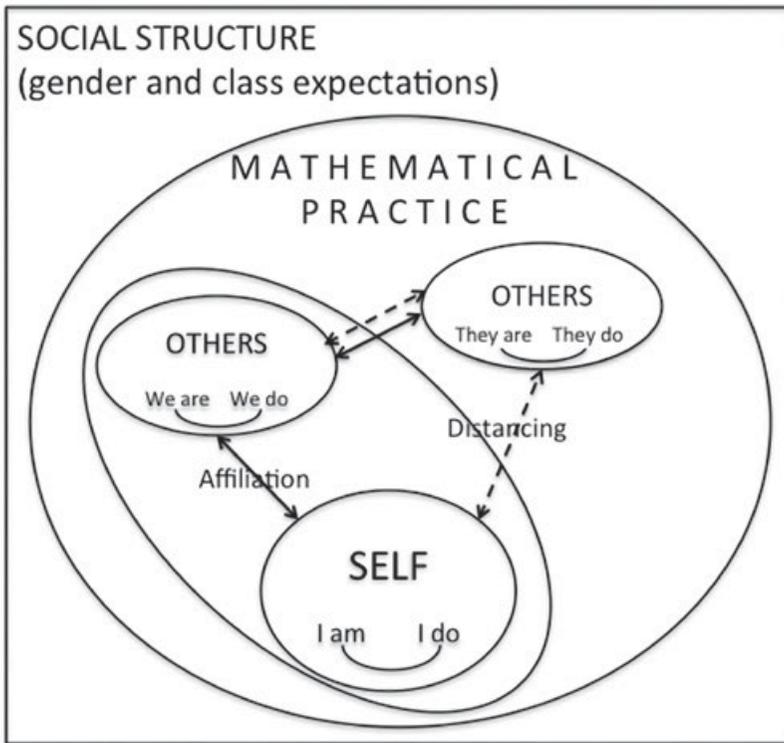


Figure 1. This model of MI shows how one student's identities (both narratives and acts) are constructed in interactions with other peers in a particular peer cluster and in a particular social context.

this country has seen a dramatic increase in the incorporation of women in formal education and the workforce over the last 50 years (Esteve & López-Ruiz, 2010), important gender differences still persist in Chilean society. Participation in the workforce is still at less than 50% of the female population compared with 70% of the male population (Instituto Nacional de Estadísticas, 2012), wages for women are estimated to be 30% less than equivalent wages for men (Fuentes, Palma, & Montero, 2005), and working women are still highly concentrated in areas of service and trade (74%) and occupy less competitive jobs with lower salaries (Voz de Mujer, 2013). In addition, traditional gender roles predominate. It is still commonly believed that the most important role for women is to stay at home and take care of their families, with women tending to leave the labor market after having children or when their partners earn enough money to sustain their families (Stuven, 2013). In contrast, the most important role for men is to go out and work (Voz de Mujer, 2010). These traditional roles are adopted more readily by women from lower socioeconomic groups, which have lower levels of female participation

in the workforce (Casen, 2013) and a higher percentage leaving the workforce after having children than women from higher socioeconomic backgrounds (Stuven, 2013).

Participants

This study was intentionally situated in a school that represented an average school for the working-class population in Santiago de Chile: a privately subsidized school with average achievement on the national census of evaluation, Sistema de Medición de la Calidad de la Educación (Education Quality Measurement System; SIMCE). The current Chilean policy allows privately subsidized schools to be selective in admitting students and to charge a small tuition fee. Consequently, these schools are highly homogeneous in terms of the socioeconomic status of their students (Mizala & Torche, 2012). Families from this school can be classified as lower working class (AIM Chile, 2015), meaning that most adults did not finish secondary school and work in services and manual labor. Furthermore, despite the high level of socioeconomic homogeneity, a policy of mixed ability means that these schools contain high variation in students' achievement inside the classroom (Ramirez, 2007). The school that is the focus of this article was considered by teachers and students to be a demanding school in which students were expected to reach a 70% mark in order to pass each grade up to Year 8 (14 to 15 years old). Students who did not reach this score were held back a year.

This study focused on a Year 7 (13 to 14 years old) mathematics classroom with 39 students. The teacher (Ms. P) had 2 1/2 years of previous experience teaching mathematics. She had completed a general (not mathematics-specific) teacher training course at a nontraditional university and had relatively limited experience and preparation in mathematics. Despite her relative inexperience, Ms. P was considered by the head of the department and her students to be a very good teacher.

The three girls on whom this study focuses (pseudonyms "Maria," "Carla," and "Katia") were intentionally selected to represent girls who were developing positive relationships with mathematics but who belonged to different peer clusters inside the classroom. All three girls were among the highest achieving students in the classroom, and all of them reported in an initial survey that mathematics was their favorite subject. In addition, all of these girls were mentioned by their teacher and by some of their classmates as students with a good relationship with the subject.

Data Collection

Data collection was performed during the second semester of the academic year between September and December 2013 (in Chile, the academic school year begins in March and continues through mid December). To gain knowledge of the classroom and cultivate the trust of the students, the first author visited the class regularly and participated in their mathematics lessons and some of their class meetings. During these visits, she walked around the classroom and frequently talked with all of the students. Four observation and interview protocols were

either designed or adapted in order to consider both social and psychological (subjective) perspectives in the analysis of students' identities (Cobb, Stephan, McClain, & Gravemeijer, 2001).

Lesson observations. The purpose of the lesson observations was to describe the predominant mathematical practice in the classroom and how students' participation was exercised and distributed. Observations were made throughout the semester. Field notes and audio recordings were collected. Low-inference information (every public intervention that each student made in public class discourse) was also gathered from four lessons of the unit on Data Management and Probabilities (the final unit of the year).

Mathematical Groups Interviews. The purpose of this instrument was to identify the different *kinds of mathematical students* perceived in the classroom, identify how mathematical competence was understood, and explore where and how each of the three girls positioned herself and was positioned by others in relation to these groups. Students who participated in these interviews were sampled from the whole class to represent different levels of achievement. A total of 16 students were interviewed both individually (seven boys and nine girls) and in groups (a group of seven boys and a group of seven girls). In addition, the teacher was asked the same set of questions during an individual interview. All participants were asked to group students by answering the following question: "Are there groups of students that show a similar relationship with mathematics?" They were asked to build as many groups as they could. It was explicitly stated to them that students in a group should be similar within the group and different from other groups. After completing this task, all participants were asked to elaborate on the criteria they used to group students in order to explore cultural models in relation to mathematics.

Peer Clusters Interviews. This interview was adapted from Cairns, Xie, and Leung's (1998) protocol for mapping the classroom peer group structure in terms of naturally occurring (i.e., by peer friendship rather than academic) peer clusters. Students were intentionally sampled to represent different peer clusters. The teacher selected students for initial interviews, and information from these interviews was then used to select students for further interviews until new interviews did not provide more clusters. A total of 12 students were interviewed (five boys and seven girls). Students were asked to cluster their classmates by answering the following question: "Are there people in this classroom who hang around together a lot?" Interviews lasted about 10 minutes on average. As with the Mathematical Groups Interviews, students' responses were explored further in order to capture meanings associated with each cluster.

Mathematical Narrative Identities Interviews. Individual interviews were used to capture students' narrative MIs. Seven girls and seven boys from the classroom were interviewed following this protocol, including the three girls on

whom this article focuses. These interviews were an adaptation of two existing instruments: the Mathematical Life Story (Lewis, 2013) and Identity Mapping (Ylvisaker, McPherson, Kayes, & Pellett, 2008). The Mathematical Life Story was used to engage each girl in building a narrative regarding her historical relationship with mathematics. Each girl was asked to think about her “maths story”: how she came to develop her current relationship with mathematics and how she imagined this relationship would develop in the future. Critical moments in this story (moments in which the girls saw their mathematical relationships changing) were marked on a map, identifying highest (positive emotions) and lowest (negative emotions) points in their mathematical lives (following Lewis, 2013; see Figure 2). The Identity Mapping was employed to describe the feelings and behaviors that each girl associated with herself when doing mathematics as well as her perception of how others saw her while she was doing mathematics. After talking about these feelings and behaviors, each girl was asked to think of a name (a mathematical character or metaphor) that portrayed her unique relationship with the classroom mathematical activity (see Figure 3). Both the Mathematical Life Story and the Identity Mapping lasted about half an hour. During the interview, the researcher encouraged the girls to think of people who influenced their stories and their current relationship with mathematics. In other words, the interviews focused on how others had shaped or were shaping these girls' MIs.

Data Analysis

Mathematical practice. Through analysis of the lesson observations and interviews, we sought to understand how mathematics was described and practiced in this particular classroom, unveiling the participation structures in place, students' positionings within these structures, and how competence was constructed. For the analysis of participation structures, lesson observations were coded in terms of (a) lesson activities and tasks in order to describe lesson structures (Wells, 1993) and (b) students' participation in classroom public discourse (any utterance that was somehow accepted or acknowledged by the teacher). For the first type of coding, activities and tasks were identified, their duration was recorded, and they were described in terms of how the activity was structured (whole-class discussion, individual or peer group work, and teacher guided) and its purpose (introduction of new content in the unit, formalization, revision, and reinforcement). For the second type of coding, the first author made notes during lesson observations on every public participation of each student in whole-class public discourse (e.g., the question or affirmation that was made and who made it). Students' private interactions were not coded. A seating plan with students' names was used to mark each student's response to all of the teacher's questions. All spontaneous student interventions and signs of disruptive behavior were also marked on the seating plan. After the lesson, the seating plans with the notes were compared with the audio recordings, and each interaction (usually a question from the teacher) was coded regarding what response was expected

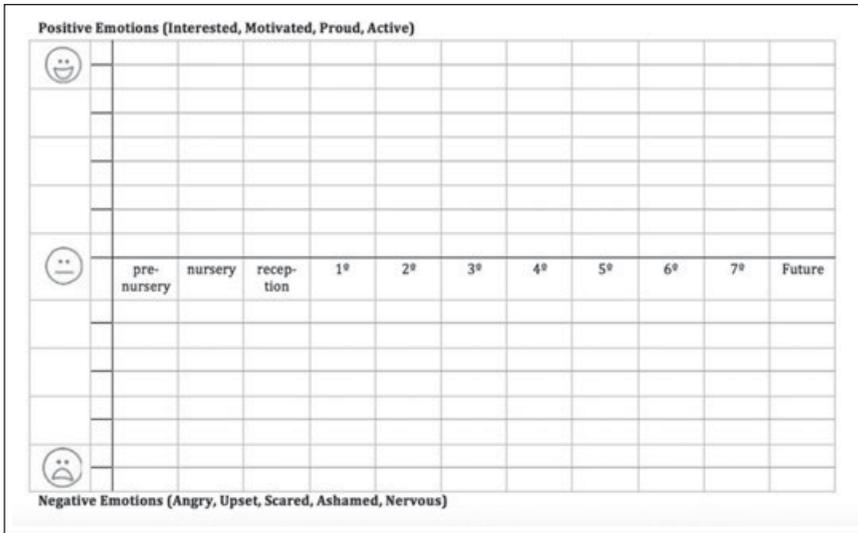


Figure 2. For the Mathematical Life Story instrument, each of the girls was asked to tell her mathematical story and to mark her highest and lowest points on the graph. Numbers indicate grade levels.

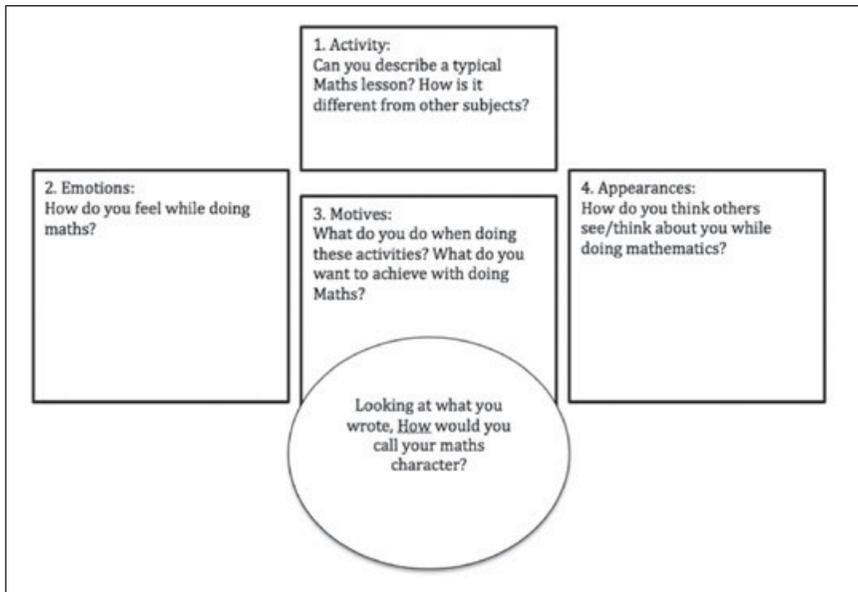


Figure 3. For the Identity Mapping instrument, each girl was given this instrument without prompts. Guiding questions are inside the squares.

(considering the response given and if it was accepted; for similar approaches see Hiebert et al., 2003; Radovic & Preiss, 2010; Wells, 1999). These codes allowed us to identify open questions (often personal experiences, opinions, and justifications) and closed questions (usually defining or naming a concept, analyzing an example or information given, or applying a concept or procedure to an example) and how they were distributed among students in the classroom.

In addition, in order to analyze what notions of competence were in place in the classroom, data from the students' Mathematical Groups Interviews were coded thematically, identifying criteria to differentiate groups (predominant cultural models in relation to mathematics). This coding procedure was highly driven by the data, and we followed an iterative method and the constant comparison analysis suggested by Strauss and Corbin (1998). Particular attention was given to the gendered aspects of groupings and cultural models used by students.

Map of the peer culture. Different peer group clusters were identified using the Composite Social Maps software (SCM 4.0). By including information collected in the Peer Clusters Interviews (each group reported by each student interviewed), the software computed a co-occurrence matrix and then identified groups in terms of similarities between co-occurrences of individuals using a correlation matrix (Cairns, Garipey, Kinderman, & Leung, 1996). After groups were identified, a new matrix was created that contrasted descriptions that the interviewees gave about their own and other groups. With this information, a general account for each peer cluster was constructed.

Girls' MIs analysis. In line with the theoretical underpinnings of this study, we considered identity in a dialectical process of development that can have moments in various forms: stories, names, figures, models, ways of participating and relating in practice, and so forth. To address this complexity, different data sources were organized into a single case document (one document per girl). This document included (a) a description of their individual participation in whole-class public discourse (lesson observations), (b) their own definitions of competence (Mathematical Groups Interviews), (c) their own positioning regarding these definitions (in which group they classified themselves), (d) their definitions of membership in their peer clusters and how they distanced themselves from others (Peer Clusters Interviews), and (e) their interviews regarding their mathematical stories and relationship with mathematics in the present and imagined futures (Mathematical Narrative Identities Interviews). In addition, information from other students and the teacher (quotes in which each girl was mentioned) were included in the document as a way of gaining access to how they were positioned by others in classroom discourse and practices. These data were transcribed in Spanish, and relevant quotes were translated by the first and third authors (who are fluent in both English and Spanish) and reviewed by the second and fourth authors (who speak English as their first language).

Each document was first analyzed individually, and only later were comparisons between the three girls explored. Comparison between different sources of data was central, in particular for the identification of potential tensions and conflicts. A tension was regarded as an opposition between different aspects of MIs (e.g., self-positioning versus others' positioning, narrative versus acted, past versus present versus future), and conflicts were identified when these tensions were associated with negative effects. Tensions and conflicts were analyzed, exploring the identity work that the students did in their narratives, particularly what cultural models students used in trying to resolve conflicts.

Results

This study aimed to contribute to the literature on MIs by exploring in detail the singularity of experiences of three girls who were seen, and saw themselves, as successful (high achievers) and engaged in their compulsory mathematics classroom. We present the main results of this study as follows: First, we offer a general account of the predominant mathematical practice in the classroom and how gender was involved with this practice. Second, we describe the process of developing MI for the three girls on whom we chose to focus.

Mathematical Practice

The analysis of the lesson observations and students' Mathematical Groups Interviews suggested a traditional pedagogical practice (see, e.g., Boaler, 2002a) in which curriculum, content, and interactions were highly controlled by the teacher. New content was introduced gradually, with large amounts of time being dedicated to rehearsal of previous content. The teacher commented that she followed this gradual approach because she was concerned with making mathematics accessible for all students in the classroom, especially those who found it particularly difficult.

In terms of participation structures, most whole-class public discourse was organized following the initiation-response-feedback (IRF) pattern (teacher's question \rightarrow students' answer \rightarrow teacher's feedback; see Mehan, 1979; Sinclair & Coulthard, 1975). Most teacher elicitations (63%) were identified as closed questions (students were expected to find a correct answer), with only 22% being open questions. Most lessons revolved around memorization of concepts and algorithms, which students used to solve simple routine problems. In addition, a large variability was found in students' participation. Although most students participated on only a few occasions (25 students participated only once or less per lesson), others were considerably more active (e.g., one of the focus girls, Carla, averaged close to seven questions per lesson; see Figure 4).

Different Kinds of Students

Three different kinds of students were identified from the Mathematical Groups Interviews: the *Effortless* group, the *Effortful* group, and the *Lazy* group. One boy described these groups as follows:

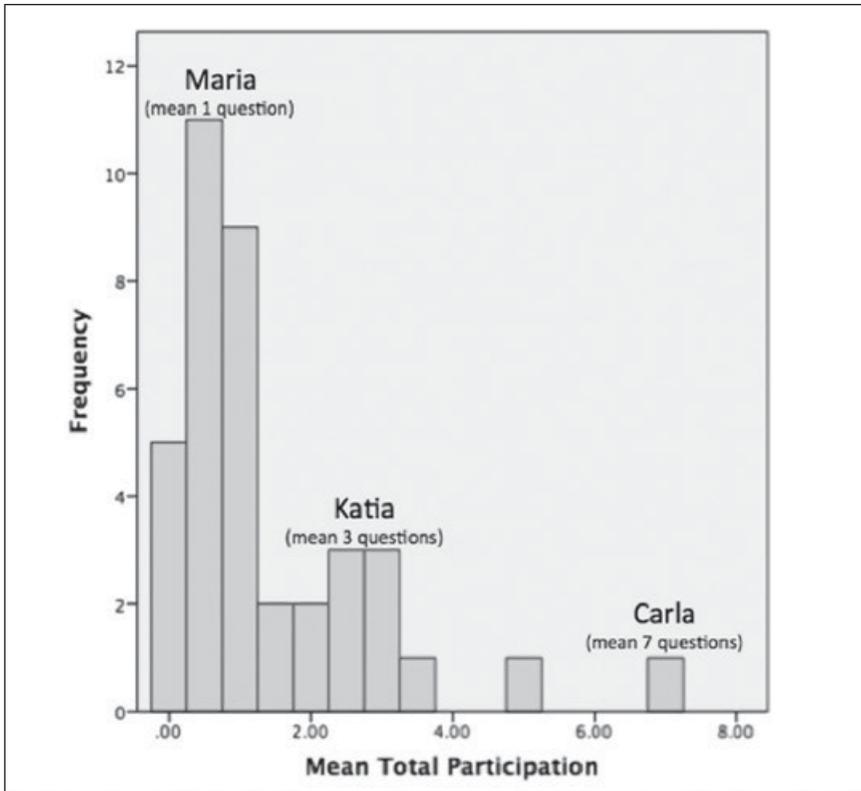


Figure 4. Histogram of mean participation in whole-class public discourse per lesson ($n = 38$ students, mean = 1.3, $SD = 1.5$ questions per lesson per student).

The first group [Effortless group] is the smart one. This group gets high grades even if they don't study, even if they don't come to class. This one [Effortful group] comes second after the smart ones. . . . In this group they have to study more and if they don't come, they get worse grades. And this last group [Lazy group] is the one that always do bad, even in sports. . . . they are the lazy ones.

The description of the Effortless group resembled the shared cultural model of mathematics as requiring natural ability (e.g., Walkerdine, 1998). Thus, because members of this group did not have to put in effort in order to achieve, they were simply "good at it." In fact, the student who was perceived by everyone as the best in the class (Christian), who belonged to the Effortless group, was often seen sleeping at his desk in the last row. In addition to Christian, three girls and another boy were consistently mentioned as part of this group.

In contrast, the Effortful group was characterized as having a positive relationship with mathematics but needing to put forth some effort in order to be successful. Students described members of this group as frequently participating

in class and showing interest. Although a couple of boys were mentioned, it was mainly girls who were identified as part of the Effortful group. When this was discussed in a group interview, two boys identified themselves as Effortful, and the rest of the boys started laughing and bullying them because of this identification. It is possible to conclude from these observations that the Effortless position was regarded as an imperative for high-achiever boys in this classroom but not for girls, who could position themselves as either Effortless or Effortful even if they were actually high achievers.

Finally, the Lazy group was consistently described as being composed of low-achiever boys who exhibited a bad relationship with mathematics and who did not make an effort to achieve. Most of the interviewees (including some of the students who self-identified as Lazy) regarded this group's lack of effort as due to laziness and lack of interest. Three girls, friends of these boys and part of the Adolescent group (see below), said that the boys' lack of achievement was not related to lack of effort but to an actual difficulty in understanding and staying focused during lessons.

In addition to this gendered dimension in the definition of different kinds of mathematical people in the classroom, a relationship between mathematical groups and peer clusters was observed. In particular, the peer clusters to which the three girls of this study belonged (the Adolescent group, the Normal group, and the Korean group) were composed of contrasting mathematical people. The Adolescent group contained most of the boys who were identified as being in the Lazy group (four out of five) and all the girls who were regarded as part of the Effortless group (three out of three), including Maria. The Korean and the Normal groups were mixed in terms of achievement and participation. High achiever girls from these groups (i.e., Carla, Carla's friend Daniela, and Katia) were considered Effortful by their peers (see Figure 5).

Girls' MIs in the Classroom

The three girls who are the focus of this study belonged to three different peer clusters and identified with their respective groups' identities. In addition, they contrasted largely in how they were viewed as mathematical people by their peers, how they participated in classroom public discourse (acted identities), and the cultural models they identified with and used in their narrative MIs. These differences are summarized in Table 1.

Maria: Balancing mathematics and “the normal life.” According to data from the Peer Clusters Interviews, Maria belonged to a peer cluster of four girls and four boys named the Adolescent group. This group was seen as popular because “they acted as if they were older, thought they were the coolest, showed off about what they owned [clothes or technology], and appeared not to be especially concerned about school.” Some classmates from other groups mentioned that girls in this group were too preoccupied with their appearance—wearing skirts that were too short, using makeup, and always hugging and kissing each

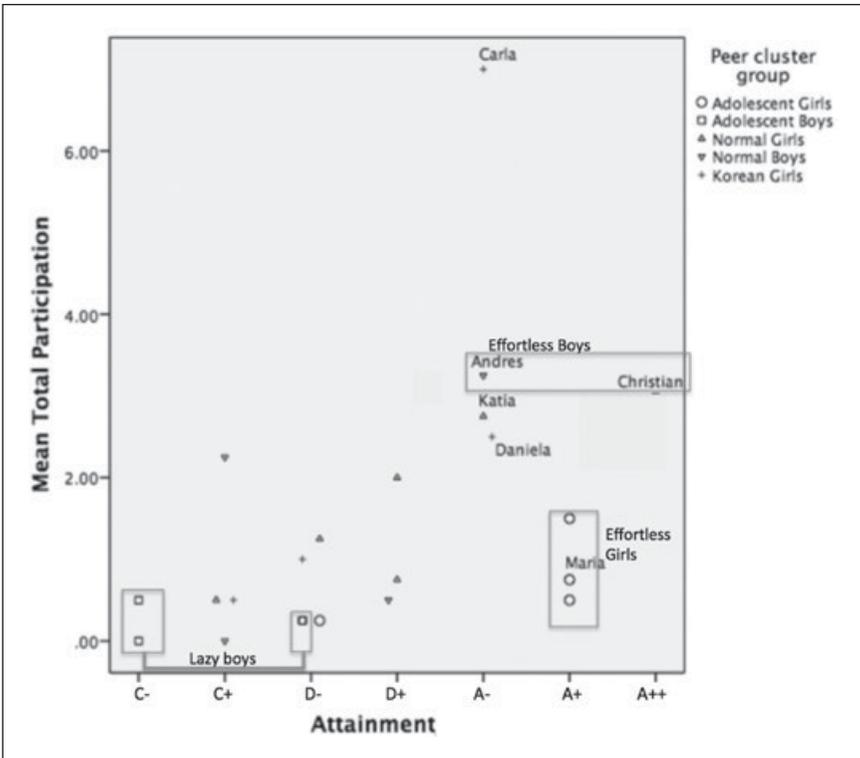


Figure 5. Student participation in whole-class public discourse (as measured by observations) and achievement (as measured by grades). Only focal students (Maria, Carla, and Katia), their peer clusters, and students who were consistently named as “Effortless high achievement” and “Effortless/Lazy low achievement” were included ($n = 23$). The figure shows how Effortless girls were clustered with Lazy boys (in the Adolescent group—Girls \circ , Boys \square) and how the Normal group (Girls \triangle ; Boys ∇) and the Korean group (+) were diverse in terms of achievement and participation.

other. Some of them, they said, even had boyfriends. The boys in the group were described by all of their classmates as exhibiting low achievement. Peers from other peer clusters attributed this low achievement to laziness, but the girls in the Adolescent group attributed it to low ability. This low achievement was persistent in these boys' mathematical stories; three of them had been held back a year or more. During the interviews, boys and girls from the Adolescent group described themselves as “the most mature group in the class.” They commented that other clusters (especially the Normal group) behaved childishly.

Data from classroom observations showed that Maria and her friends (both female and male) rarely participated during whole-class public discourse (see Figure 5). She was never observed offering an answer or raising her hand. Her participation was limited to direct questions from the teacher, never engaging in

Table 1
Summary of Case Study Girls' Identities

	Maria	Carla	Katia
Peer Cluster	Adolescent (mature, hyper-femininity, popular, social)	Korean (weird, loyal)	Normal (girls are quiet, and boys are loud and act like kids)
Main Acted Identities	Detachment	Protagonist and challenging	Compliant and support-seeker
Mathematics Group	Effortless	Effortful	Effortful
Mathematics Cultural Model	Mathematics as a natural ability	Mathematics as different	Mathematics as male
Main Narrated Identities	Efficient	Different	Responsible
Valued Forms of Doing Mathematics	Independent and collaborative	Wider and complex	Straightforward and procedure oriented

prolonged exchanges with her. When Maria and her friends were observed doing peer–individual work, they worked together as fast as they could in order to finish the mathematical activity and then engage in off-task conversations. We interpreted this acted identity as a detachment from the mathematical practice.

Maria's observed behavior was highly consistent with how she was described in the Mathematical Groups Interviews by her peers. She was frequently mentioned as an example (by 13 of 16 students) of the Effortless group for whom "mathematics comes easy," like a natural ability, a position that she assumed with no conflict:

I am in the first group and the three of us [two female friends] are in this group, because I think we are the ones that understand the most. . . . Ah, and Christian is also in this group. I think we . . . I don't study, I just pay attention during lessons and it gets recorded in my head, but I don't study.

The absence of conflict in Maria's account is also clear in how she described her emotions when doing mathematics. She did not describe strong positive feelings, despite naming mathematics as her favorite subject. On the contrary, she mentioned emotional states related to the absence of discomfort, using words like *comfortable*, *tranquil*, and *placid*: "I feel comfortable because I understand, because if I didn't understand I wouldn't probably feel comfortable."

When constructing her Mathematics Life Story, Maria stated that this sense of tranquility had accompanied her throughout her school life. However, she commented that, during the last few months, these feelings had changed slightly and that she had felt "less positive about it . . . or less worried about it." She linked

these feelings to the fact that, to her, mathematics (and school) had lost significance in her life because new activities had acquired more relevance:

What is happening is that I don't like to keep only with school, because when I was younger I cared only about school. But now I would like to do sports, or participate in a workshop. . . . I'm the president of my class and I did swimming, but I left it because so much swimming was making me look bigger. Now I'm also dating, so things are changing. School is still important, but I don't know if it is adolescence or what, but it is like you start to feel lazy.

It is interesting to note here that new activities appeared to be somehow gendered: She was starting to date, and she had to choose activities that allowed her to be feminine and beautiful (and not "big").

It was the tension between different activities (socializing versus doing mathematics) that appeared to shape Maria's definition of her mathematical character in the Identity Mapping. When discussing which name she would use to describe her mathematical personality, she chose to be called "efficient":

That's why I try to be quick doing the exercises, because if I do that I can think in maths for a moment and then I can return to the "normal life" [Both laugh]. . . . That's what I like to do . . . to do my work and then relax . . . I'm gonna write efficient here as the name of my character.

Consistent with her group's way of participating and her desire to balance mathematics and her social obligations, Maria valued a certain freedom given in the class to work independently and in collaboration with her peers:

Ms. P gives some time for doing the sheet and this time is like a little free time because you can. . . . I mean, not a free time in a bad sense, but you can work with your desk partner, or talk the exercises with your group . . . that's dynamic and that's why I like maths more than other subjects.

The links between Maria's mathematical positioning as an effortless high-achiever girl and her belonging to a particular peer cluster in the classroom could be seen in two different ways. First, she defined her positioning as effortless in alignment with her female friends, as noted by the use of "we" in her description of the Effortless group above and by how she shared activities during mathematics lessons with them (to work and to chat). In addition, it can be hypothesized that Maria's relationship with the boys who belong to her peer cluster (the Adolescent group) was also an identity resource in two different ways. First, she presented her negotiation of effort as part of growing up and being "mature." Because the boys in this group were older than the rest of the class, their friendship supported the girls' notions of being mature. Second, by positioning the boys in her group as "having difficulties" (as opposed to being lazy), Maria did not recognize the cultural model of "natural ability as male," allowing her identification with an effortless identity without perceiving it as conflicting with her female identity.

Finally, it is interesting to note that the metaphor of "balancing" was also present in Maria's account of the future and how she foresaw her relationship with

mathematics. Here, for the first time, the tension that she was living with became a conflict because she believed she would need to lower her expectations in order to balance mathematics and life. This is exemplified in her shifting from aspiring to study engineering to administration:

Maria: Engineering is like . . . everything is mathematics, so I don't want to start something for two semesters and then not being able to pass anything. . . . I don't want to get stuck.

Interviewer: Why do you think it is going to get this difficult? Have you ever felt this kind of difficulty in mathematics?

Maria: No, never, because I have always had good grades, so for now I am relaxed, but I don't know if it's gonna be like this forever and I don't want to end up in a bad university and I also don't want to end up only doing mathematics all the time, because I like it but not that much, so administration is a balance between life and mathematics.

Interviewer: And how do you see life in the future?

Maria: I wouldn't like to leave home too old, so my idea is to finish university, have a stable job and then leave . . . or maybe leave before, but go outside . . . into the real world, start my life . . . in general it is like living alone. I mean, I don't expect to marry too young, no grrrr, maybe marrying, yes, but not young, and not having kids young . . . it is like having everything that normal people have. . . .

Interviewer: What you mean normal people? Does this happen with the people you know?

Maria: Yes and no . . . I know people that do this, but for example my mum has always been dependent and I don't want to be like this, I don't want to depend on nobody.

As the transcript above suggests, there is a gender dimension in how Maria envisions her future. Compared with her mother, she would like to challenge the traditional role of a dependent woman and wife by being a young independent person. Independence would require a career, but the career would have to allow her to have this independent role before taking on the wife and mother role; thus, it would have to be quickly acquired and not a demanding engineering career.

In summary, Maria appeared to be developing an MI which had high consistency between the way she acted, how her classmates positioned her in the classroom, and how she perceived herself. All of these different sources of identification served as resources for an emerging MI based on the cultural model of mathematics as a natural ability. The resources provided by her classroom mathematical practice (which did not challenge her) and her relationship with her friends (aligning with her female friends and positioning her effortless engagement as consistent with her group maturity) seemed to allow Maria to experience this as a positive MI that contributed to her feeling comfortable when doing mathematics.

The main tension to manage in maintaining this identity seemed to be related to balancing mathematics and what she called “normal life.” Although in the present, this normal and gendered life involved being social (e.g., partying, dating, hanging out with friends), in the future it seemed to be related to becoming independent (e.g., living alone before marrying and having kids). Although her balancing of mathematics and normal life was not experienced as a conflict in the present, she anticipated the future as being rather challenging because she would have to manage her expectations in order to be able to cope. The potential conflict that she anticipated appeared to suggest some form of contradiction between pursuing a demanding career involving mathematics and, at the same time, conforming to the gender role of being a wife and a mother.

Carla: The math that I am taught and the math that I like. Data from the Peer Clusters Interviews revealed that Carla belonged to a peer cluster consisting of four girls and no boys. All of their classmates identified this group as the Korean group because members of this group liked Korean and Kitsch Pop music. Because they were the only cluster in the classroom that liked this type of music, they were described as “weirdos,” and some of their classmates said that they were sometimes bullied because of this. However, one of their classmates said that the Korean group girls did not care if they were bullied: “They don’t mind—they feel strongly about liking Korean music.” The girls themselves did not talk much about their interest in music; for them, the important thing about their group was that they were “inseparable” and that they did lots of different things together. They said that they cared about each other. One of their classmates said that if someone said something to one of these girls, the others would defend them, especially Carla, who was described as the leader of the group.

The analysis of classroom observations showed that Carla actively participated during whole-class public discourse (see Figure 5). Every time there was an episode of multiple questions and answers between the teacher and students, she contributed with answers. Furthermore, Carla was one of the few students who made spontaneous contributions to the public discourse. On a few occasions, she commented on what the teacher was saying or answered a question with an unexpected answer. Thus, Carla’s participation was rather different from that of her classmates and from what her teacher was sometimes expecting. This constituted a main acted identity that we characterized as *protagonist and challenging*. The result of such behavior was that it forced the teacher to transform her usual evaluation follow-ups into more extended interactions. In addition, in some of her interventions, Carla risked giving incorrect answers (which were uncommon in the classroom) that led the conversation with the teacher into more extended episodes.

Carla’s behavior made it more difficult for her peers to categorize her as a particular kind of student in the Mathematical Groups Interviews. Although she was mentioned by four classmates and her teacher as having a positive relationship with mathematics, this positioning was not associated with ability but with interest and

participation. However, because students commonly linked participation and interest with effort, it was difficult for them to recognize Carla as a “good mathematician”:

For example, Carla supposedly knows a lot, but then in tests she doesn't do as good, or sometimes she makes silly mistakes. Maybe she gets nervous or she does not care about making these mistakes.

Carla's self-positioning in relation to this public identity was rather positive and somehow not conflicted. She perceived that her relationship with mathematics went beyond “being right” or having to use and apply “one main technique.” In this sense, Carla described herself as different compared with her classmates:

I feel good when doing mathematics, not like the majority who feel neutral about it. We see mathematics as wider, not like others who have one main procedure and just apply it. I have more techniques and I reflect more when doing mathematics. Mathematics has certain confusion that I like . . . it is more like understanding it than simply applying a procedure.

It seems that, unlike her classmates, Carla did not relate her participation and behavior to effort but to engagement in an activity that went beyond obtaining the correct answer. This perception shaped the way that she described her relationship with mathematics in her Mathematical Life Story in which she related an increasingly positive relationship with mathematics that was extremely easy at the beginning but became increasingly complex with time. She described the most negative moment in her Mathematical Life Story as experiencing a problem with how mathematics was taught to her during Years 3 and 4 (ages 8–10):

The teacher was too square; she did not give us the opportunity to open up to mathematics. She used to give us a technique and we had to apply it. But then in Year 5 I felt that my mind opened up. . . . I had to remember contents from previous years and in order to think about contents that were going to come in the future.

In the present class, she said that Ms. P's lessons gave space for developing ideas, engaging reflexively, and understanding a type of mathematics she valued because it was complex and wider: “And this year I am with this teacher, and if I give her an idea, she says yes, but you have this error, and she helps me to develop my own strategies with mathematics.”

It could be argued that Carla used spaces offered by the teacher in the classroom to develop the kind of mathematics that she liked. In this sense, she considered that this kind of engagement was partly the student's responsibility: A student can either comply with repetitive practice (which she linked with some sort of disengagement) or use the spaces provided (as consequently observed in her forms of participation).

Carla did not make direct links between her MI and other members of the Korean group. However, she appeared to use the shared peer cluster identity (“being different” or “being weird”) to support the way she preferred to participate when doing mathematics. One girl from the Korean group, Daniela, offered a

compelling example of how belonging to this peer cluster could function as a resource to sustain its members' MIs:

Sometimes it makes you feel anxious when the teacher asks you something because some of the guys bully you and that makes you feel stressed. For example, if this happens to one of them, they don't say anything. For example, if Mario [low-achieving boy from the Adolescent group] says something silly, you have to stay quiet because if you didn't all the class shouts "forever alone,"¹ even if you shouted with your four friends, they still shout you "forever alone." . . . If we don't understand something, we just ask as many times as we need. Sometimes we get bullied, but we don't care.

As with Maria, it can be argued that Carla was also developing MIs that showed high levels of consistency between her acts and narratives. She valued a "wider mathematics" that went beyond rote techniques, and she risked forms of participation that allowed her to shape practice to the mathematics that she liked. Carla therefore constructed an MI based on a cultural model of "mathematics as special or different." Two tensions were observed when she was managing these identifications: First, there was a tension between the opportunities provided by the teacher and the kinds of opportunities that Carla wanted to take. Here she actively participated and even monopolized exchanges that supported "the mathematics that she likes." A second tension was observed between Carla's secure identification with mathematics and the unclear positioning of Carla as a "good mathematics person" by her classmates. Such misalignment can be exemplified in the bullying episodes and how peers from other groups ignored her relatively good achievement. Again, however, Carla seemed not particularly conflicted by this tension, at least in the present, because she did not recognize difficulties or effort in her engagement. In light of these two tensions, it can be argued that maintaining and managing positive and good identifications with mathematics for Carla required an important amount of self-confidence, which was mainly resourced by her peer cluster shared identity.

Katia: The complicated math and the simple math. The analysis of the Peer Clusters Interviews showed that Katia belonged to a large mixed-gender cluster that consisted of six girls and four boys. Other students in the class described the girls in this group as "the quiet ones" because teachers never yelled at them for talking. Girls from the Adolescent group described these girls as childish, contrasting them with their own social self-identity as mature. One girl and one boy within this cluster named themselves as the Normal group and noted that they enjoyed childhood as appropriate to their age and did not want to grow up too fast. As one of them said, "We live what we are supposed to live." In terms of their relationship with mathematics, members of this group were highly diverse with

¹ "Forever alone" refers to a popular meme used to express loneliness and disappointment with life (see <http://knowyourmeme.com/memes/forever-alone>). Students in the classroom shouted "forever alone" in English when they disapproved of or rejected some of their classmates' behaviors or comments.

respect to grades, but none of them were regarded as lower achievers or lazy. In contrast to the girls in this group, who were described as quiet, the boys were seen as disruptive and noisy.

In terms of frequency of participation, as revealed by the analysis of lesson observations, Katia was located somewhere between the high participation of Carla and the relatively peripheral positioning of Maria (see Figure 5). In contrast to Maria and her friends, Katia usually raised her hand and offered answers to the teacher's questions, but compared with Carla, she was never seen making spontaneous contributions during whole-class public discourse. She sometimes responded to questions that required an opinion or justification; however, her answers were much more closely related to what the teacher was expecting or what had previously been said, an approach that we interpreted as a main acted identity of compliance with the teacher's pedagogy.

Another feature of Katia and her friends' participation was evident during peer-individual work and strongly contrasted with Maria's detachment. Katia worked with her desk partner but was also very active in looking for support. She was frequently observed raising her hand, calling the teacher to check on her answers, or asking other students.

In the Mathematical Groups Interviews, four classmates and the teacher described Katia's positive relationship with mathematics as a product of her interest and participation but not ability. Like Carla, Katia was seen as having to put in effort in order to achieve. Although it could be argued that Carla and Katia were offered the same position in terms of how their peers and the teacher saw them in the classroom, their response to this positioning was rather different. In contrast with Carla's comfortable identification, Katia presented a more conflicted experience: She found it difficult to see herself as part of the "smart" group (i.e., the Effortless group) because she perceived herself as slow and without intrinsic ability, a trait that, according to her, only a few males had:

I'm in this group, but it is not like we are fast, because for example, Christian, he is like the smartest in the class . . . in general, he is good at everything . . . it is like in the personality of each of us. For example, Andres [one of the boys from her peer cluster], I see he has been continuously talking and I ask him if he did his work and he has it all done and he is even doing the second one and we haven't really finished the first one.

To solve this conflict, Katia positioned herself and the girls of her group as "the responsible ones" because they placed work before social interaction. She also linked this behavior with maturity and offered a gendered opposition to justify her behavior:

I think I am responsible, and I think it is related with maturity because you need to be conscious of what is going to happen, you need to be responsible and do what you have to do. . . . Women are more professional, like more focused. Boys are not mature enough. It is like they are not aware of causes and consequences. But women it is like "no . . . if I don't do it I would have a bad grade so I am going to do it and I'm gonna [be] responsible."

Another key aspect of Katia's MI was her experience that mathematics was often presented as difficult and complicated, but, in reality, it was simple and straightforward. To Katia, learning mathematics involved applying a method taught by the teacher and then waiting for the teacher to offer a new method if the first one was inadequate or too challenging.

I would like the teacher to teach us techniques that weren't difficult so we could understand them in the moment and then we could move quickly, like the magic formula, easy procedures to understand. . . . But in mathematics the teacher is like too serious and she explains everything as if it is very complicated.

Katia's positionings were clearly framed by her relationship with her peer cluster. She shared a dutiful and compliant approach to mathematics with her girlfriends and contrasted her approach with that adopted by the boys. Although she proposed this as a generalization (boys versus girls), she used one of the boys in her group (Andres) as an example. To her, Andres represented the idea that you don't need to be responsible in order to achieve. In addition, she assumed responsibility in terms of helping those friends who (as boys) were not responsible and did not have the ability required to be effortless: "I think we are four girls and we will have to be with the boys and we will have to help the boys to focus, we will have to be responsible, because I want to get a good grade."

In summary, Katia's positive relationship with mathematics was populated with tensions that were leading to conflicted identifications with cultural models available in the classroom. Although Katia was trying to maintain the positive relationship that she declared, she failed because she did not fit the available cultural model of mathematics as a natural ability. Katia managed this conflict by displacing the source of difficulties outside. It was not only she who struggled in mathematics but all women, and she struggled because the teacher was not able to simplify math enough. In other words, the relationship that Katia had with math and the emotional states that emerged from it were heavily mediated by gender oppositions and by the teacher's scaffolding. This positioning was supported by her relationships within her peer cluster, by her alignment with her female friends and with women in general as responsible for others, and by her opposition to the immature, irresponsible male friends in her peer cluster.

It is possible that by constructing a view of mathematics as something simple, Katia was trying to gain some sense of control over the activity, thus also enhancing her sense of efficacy. She could be an "effortless good mathematician" but only when mathematics was stripped of its unnecessary complexity. However, because the mathematics offered by the teacher was not straightforward, her expectation was not met, and she found herself somewhere in between a "good at mathematics student" (who has it in himself or herself and therefore does not need support) and a "good compliant student" (who needs support and confirmation), types that apparently do not fit together in this classroom.

Conclusion and Discussion

The main purpose of this article was to systematically study three girls in compulsory education who presented a positive relationship with mathematics. This article addresses three relevant gaps in the existing literature on MIs: a focus on girls' engagement and positive identifications with mathematics in compulsory rather than postcompulsory mathematics, an interest in their developing narratives about themselves in relation to mathematics (a methodology also neglected in compulsory education), and an exploration of how this identification process is negotiated in relation to the peers and peer clusters that appear to be so significant in early adolescence.

In general, the findings of this study confirm that among girls who share a positive relationship with mathematical activity (a high level of achievement and engagement with the practice), there is an important variability in how their MIs are experienced in the classroom. Although all three girls recognized some level of tension in managing their positive identification with mathematics, the nature of these tensions and the level of conflict attached to them were very different in each case. Maria and Carla experienced positive emotions when doing mathematics and showed confidence and consistency between the way that they acted and the way that they talked about mathematics. In contrast, Katia seemed more emotionally conflicted, trying to understand what was expected of her and how to deal with the unfair situation of being effortful but not successful. In addition, as shown in Table 1, the three girls valued different forms of doing mathematics (independent and collaborative; wider and complex; and straightforward, procedure-oriented mathematics), showed different acted identities while doing mathematics (detachment or engagement on the margins of practice, protagonist and challenging of the teacher's practice, compliant and support seeking), and positioned themselves differently when narrating their identities as mathematicians (effortless or efficient, different, responsible). This confirms that, if we are to understand marginalization from mathematical practices of any population, studies need to consider diversity within groups and resist essentialist accounts (Gutiérrez, 2013).

One conclusion that can be drawn from these three cases is that peer relations had a central role in mediating each girl's MI. Their participation in classroom activities made them cognizant of their relations with other girls from the same cluster, and their peer cluster identity was used as a resource for supporting their MIs (and vice versa). These results provide further support for the argument that MIs are intertwined with students' peer relations and the peer group culture (Gholson & Martin, 2014). This is also consistent with the theoretical models that consider engaging in peer relations as a leading activity in adolescence (El'konin, 1971; Karpov, 2003) and a key source for identity development (Erikson, 1968). According to El'konin (1971), peer group relations lead adolescents' development as they pave their trajectories to adult life. Although for some girls these relationships will be rather strongly gendered (e.g., seen as sexual relations or as

caring or mothering), for others they will be seen in other ways (e.g., as being different; for example, liking Korean music).

Another conclusion of this study relates to the rather dominant presence of the cultural model of mathematics as requiring natural or innate ability in this classroom. Effortless achievement, something innate or natural, has recently been described as a common aspiration of many students in schools (Jackson, 2006). Some authors have suggested that the concept of mathematics as requiring a natural ability may be valued among students because it helps in negotiating and maintaining popularity inside the peer culture (Francis et al., 2010). As a consequence, students can either identify themselves as “natural mathematicians” (which requires recognition from others) or attempt to find alternative ways to negotiate success with popularity and belonging in the peer culture. Whereas identifying with an Effortless position provided access to status in the peer group (i.e., Maria’s example of being popular with her popular friends), finding an alternative MI required negotiating peer relations on the margins of the classroom peer group structure (i.e., Carla’s example of being different) or in opposition to those who seem effortless (i.e., Katia’s example of her relationship with Andres as an opposition between being effortless and being responsible). Finding these alternative MIs appeared to be more conflicted than the relative alignment associated with Maria’s Effortless position.

It is interesting to note that the observed relationship between the cultural model of mathematics as requiring natural ability and gender differs from other studies that focus on postcompulsory mathematics education. Although the postcompulsory literature has described this cultural model as one explanation of girls’ disidentifications from mathematics—because it is linked with being male (Mendick, 2005a)—data from this study suggest that, in this specific compulsory mathematics classroom, the effortless achievement model is also a possibility for girls’ identifications. One possible explanation for this discrepancy may well be that mathematics can have different meanings in compulsory and postcompulsory education. This case study suggests that, in a compulsory mathematics classroom, it is possible for students to see mathematics as something that is not masculine because it is not marked as different or special (Damarin, 2000). In some way, the teacher of this classroom is trying to make mathematics available for all, and she achieves (or tries to achieve) this by slowing the pace of the lessons and lowering the difficulty of the mathematics that is taught.

With respect to gender, the data from this study suggest that gender appears to mediate girls’ relationships with mathematics in at least two ways. First, through interaction with peers, these girls are preparing themselves for future (gendered) relations that influence their relation with mathematics “in the moment” in classrooms, as well as their choice of mathematics as a career path—peer groups are not only a space of belonging but also a space for becoming (El’konin, 1971; Karpov, 2003). Maria is an example of how these peer relations are gendered and how this may have an effect on her aspirations for the future: Balancing her “efforts” and work was important in showing that she could acquire femininity

for future gendered practices like dating, partying, having several different interests, and looking pretty. Her aspirations also suggest that this balancing involves adjusting her expectations in the future and therefore adjusting her role and place in the social world. It can be argued that through gendered peer relations, Maria is exercising social roles and working out where she belongs and where her place is in the Chilean gendered labor market and society. Mathematics and mathematically demanding careers become places where she feels she won't belong or where she won't be able to achieve her (gendered) goals. Her self-identification as "mature" in the Adolescent group is consistent with this consciousness of such imagined futures or places where the Normal group may be less inclined to go.

Another interesting gender-related finding from this study is that the discourse of natural ability as male seems to be a resource from which some girls might draw to negotiate their difficulties. This finding is consistent with evidence from postcompulsory education suggesting that "math as a male domain" discourse can be used not only to count girls out of mathematics but sometimes also as a resource ("having a male brain") to enter the practice (Solomon, 2012). This phenomenon was mainly observed in Katia's story, which showed a highly gendered discourse about natural ability as a male attribute. It is interesting to note that in her discourse, the effortless ones who "have it in their personality" were exclusively male (e.g., Christian or Andres), which is surprising considering that there were girls in the class who also fit this definition (e.g., Maria). This omission raises the question of why Katia adopts a gendered narrative in which men are skilled at mathematics but women need to be responsible. A possible explanation is that such a narrative has a regulatory function in Katia's identifications; by linking the cultural model of mathematics as natural ability with being male, she can sustain a positive relationship with mathematics despite not being talented ("not fast like Andres or Christian"). By attaching this natural ability to being male, she can normalize herself and preserve a positive relationship with mathematics that is not as threatening to her gender identity. In other words, it is not her problem if she finds mathematics difficult; it is simply a struggle that all women have to face. However, this discourse is not disembodied but is deeply embedded in her peer relations and is therefore resourced by her belonging to her particular peer cluster: a position that is required for the high achieving boys of the classroom, performed in practice by one of the boys of her group (e.g., Andres), and confirmed by her effortful girlfriends and by her own positioning as a "responsible and not necessarily talented girl."

With respect to the limitations of this study, we must mention the idiosyncratic nature of each mathematics classroom and how processes within them may be highly classroom-dependent. Despite this idiosyncrasy, this study provides evidence of the conceptual generalization that peer relations can be pivotal in students' development of MIs during early adolescence. Following this and previous research on peer culture and social influences on self-efficacy and self-concept (Francis et al., 2010; Marsh, 1987), we argue that future research and

summary readings of research (e.g., meta-analyses or reviews for policy) need to pay careful attention to the structure of peer clusters within classrooms.

Finally, there are a number of potential implications that emerge from these findings, specifically in relation to teaching practices in the mathematics classroom. The findings suggest that the subjective experience of doing mathematics, even among those who have a positive relationship with it, can differ in important ways between individuals. Consequently, teachers who acknowledge this variability might actively explore students' individual ideas about mathematics as well as understand how these ideas are influenced by peer relations. Students' approaches to participation in the classroom are influenced by how peer relations are intertwined with MIs. Cultivating greater awareness of this relationship may help teachers distribute opportunities for participation more strategically in public discourse and also scaffold collaboration between students during private group work (see, also, Bishop, 2012). We argue that attention to the peer group structure as it presents itself in the classroom might be significant to the formation of positive identities for girls in mathematics and should be the focus of future research in this field. The complexity revealed by this study of one classroom in a supposedly socially homogeneous school suggests that accounts of MI that ignore peer clustering might miss a crucial reference point, and methodologies should be developed accordingly.

The nested model of identity based on a case study approach used in this article enhances the methodology of previous studies, such as Choudry, Williams, and Black (2016) and Gholson and Martin (2014), in several respects, particularly in the acknowledgment of the significance of the cultural models that mediate the peer clusters and how these might afford distinct MIs. Thus, conforming to what is expected of a "normal" (gendered) identity associated with a normal peer cluster might mediate a normal, gendered MI; similarly, being "weird" might mediate an alternative MI. Acceptance of a diversity of peer clusters' ways of being and engaging in mathematics classrooms might support a variety of less obviously normal, less stereotypically gendered, and perhaps more positive MIs for girls. Future studies and thoughtful policy and practice might be wise to take these considerations into account.

References

- Agencia Calidad de la Educación. (2014). *Informe nacional resultados Chile PISA 2012* [National report results Chile PISA 2012]. Santiago, Chile: Author.
- AIM Chile. (2015). *Cómo clasificar los grupos socioeconómicos en Chile* [How to classify socioeconomic groups in Chile; PowerPoint slides]. Retrieved from <http://www.aimchile.cl/wp-content/uploads/Presentaci%C3%B3n-final-AIM.pdf>
- Atweh, B., & Cooper, T. (1995). The construction of gender, social class and mathematics in the classroom. *Educational Studies in Mathematics*, 28(3), 293–310. doi:10.1007/BF01274178
- Bishop, J. P. (2012). "She's always been the smart one. I've always been the dumb one": Identities in the mathematics classroom. *Journal for Research in Mathematics Education*, 43(1), 34–74. doi:10.5951/jresmetheduc.43.1.0034
- Black, L. (2004). Differential participation in whole-class discussions and the construction of marginalised identities. *Journal of Educational Enquiry*, 5(1), 34–54.

- Black, L., & Williams, J. (2013). Contradiction and conflict between “leading identities”: Becoming an engineer versus becoming a “good muslim” woman. *Educational Studies in Mathematics*, *84*(1), 1–14. doi:10.1007/s10649-013-9481-7
- Boaler, J. (1997). Reclaiming school mathematics: The girls fight back. *Gender and Education*, *9*(3), 285–305. doi:10.1080/09540259721268
- Boaler, J. (2000). Exploring situated insights into research and learning. *Journal for Research in Mathematics Education*, *31*(1), 113–119. doi:10.2307/749822
- Boaler, J. (2002a). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Boaler, J. (2002b). Paying the price for “sugar and spice”: Shifting the analytical lens in equity research. *Mathematical Thinking and Learning*, *4*(2–3), 127–144. doi:10.1207/S15327833MTL04023_3
- Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 171–200). Westport, CT: Ablex Publishing.
- Cairns, R. B., Garipey, J. L., Kinderman, T., & Leung, M. C. (1996). *Identifying social clusters in natural settings*. Unpublished manuscript, University of North Carolina at Chapel Hill, Chapel Hill, NC.
- Cairns, R., Xie, H., & Leung, M.-C. (1998). The popularity of friendship and the neglect of social networks: Toward a new balance. *New Directions for Child and Adolescent Development*, *81*, 25–53. doi:10.1002/cd.23219988104
- Casen. (2013). *Género: Síntesis de resultados* [Gender: Synthesis of results; PowerPoint slides]. Retrieved from http://observatorio.ministeriodesarrollosocial.gob.cl/documentos/Casen2013_Genero.pdf
- Choudry, S., Williams, J., & Black, L. (2016). Peer relations and access to capital in the mathematics classroom: A Bourdieusian social network analysis. *British Journal of Sociology of Education*, 1–14. doi:10.1080/01425692.2016.1245129
- Cobb, P., Gresalfi, M., & Hodge, L. L. (2009). An interpretive scheme for analyzing the identities that students develop in mathematics classrooms. *Journal for Research in Mathematics Education*, *40*(1), 40–68.
- Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *The Journal of the Learning Sciences*, *10*(1–2), 113–163. doi:10.1207/s15327809jls10-1-2_6
- Damarin, S. K. (2000). The mathematically able as a marked category. *Gender and Education*, *12*(1), 69–85. doi:10.1080/09540250020418
- Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics*, *93*(1), 19–33. doi:10.1007/s10649-016-9696-5
- del Río, M. F., & Strasser, K. (2013). Preschool children’s beliefs about gender differences in academic skills. *Sex Roles*, *68*(3), 231–238. doi:10.1007/s11199-012-0195-6
- El’konin, D. B. (1971). Toward the problem of stages in the mental development of children. *Voprosy Psikhologii*, *4*, 6–20. Retrieved from <http://www.marxists.org/archive/elkonin/works/1971/stages.htm>
- Erikson, E. H. (1968). *Identity: Youth and crisis*. New York, NY: W. W. Norton & Company.
- Esmonde, I., & Langer-Osuna, J. M. (2013). Power in numbers: Student participation in mathematical discussions in heterogeneous spaces. *Journal for Research in Mathematics Education*, *44*(1), 288–315. doi:10.5951/jresmetheduc.44.1.0288
- Esteve, A., & López-Ruiz, L. (2010). Union formation implications of race and gender gaps in educational attainment: The case of Latin America. *Population Research and Policy Review*, *29*(5), 609–637. doi:10.1007/s11113-009-9161-6
- Francis, B., Skelton, C., & Read, B. (2010). The simultaneous production of educational achievement and popularity: How do some pupils accomplish it? *British Educational Research Journal*, *36*(2), 317–340. doi:10.1080/01411920902919265
- Fuentes, J., Palma, A., & Montero, R. (2005). Discriminación salarial por género en Chile: Una mirada global [Gender discrimination by gender in Chile: A global look]. *Estudios de Economía*, *32*(2), 133–157.

- Gholson, M., & Martin, D. B. (2014). Smart girls, Black girls, mean girls, and bullies: At the intersection of identities and the mediating role of young girls' social network in mathematical communities of practice. *Journal of Education, 194*(1), 19–34.
- Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education, 44*(1), 37–68. doi:10.5951/jresmetheduc.44.1.0037
- Herzig, A. H. (2004). "Slaughtering this beautiful math": Graduate women choosing and leaving mathematics. *Gender and Education, 16*(3), 379–395. doi:10.1080/09540250042000251506
- Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J., . . . Stigler, J. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 Video Study*. Washington, DC: U.S. Department of Education, National Center for Education Statistics.
- Holland, D., & Lachicotte, W., Jr. (2007). Vygotsky, Mead, and the new sociocultural studies of identity. In H. Daniels, M. Cole, & J. V. Wertsch (Eds.), *The Cambridge companion to Vygotsky* (pp. 101–135). New York, NY: Cambridge University Press. doi:10.1017/CCOL0521831040.005
- Holland, D., Lachicotte, W. S., Jr., Skinner, D., & Cain, C. (1998). *Identity and agency in cultural worlds*. Cambridge, MA: Harvard University Press.
- Holland, D., & Skinner, D. (1987). Prestige and intimacy: The cultural models behind Americans' talk about gender types. In D. Holland & N. Quinn (Eds.), *Cultural models in language and thought* (pp. 78–111). Cambridge, UK: Cambridge University Press. doi:10.1017/CBO9780511607660.005
- Holmegaard, H. T., Madsen, L. M., & Ulriksen, L. (2014). To choose or not to choose science: Constructions of desirable identities among young people considering a STEM higher education programme. *International Journal of Science Education, 36*(2), 186–215. doi:10.1080/09500693.2012.749362
- Instituto Nacional de Estadísticas. (2012). *Síntesis de resultados censo 2012* [Synthesis of 2012 census results]. Retrieved from <http://static.latercera.com/20140226/1904534.pdf>
- Jackson, C. (2006). *"Lads" and "ladettes" in school: Gender and a fear of failure*. Maidenhead, UK: Open University Press.
- Kaiser, G., & Rogers, P. (1995). Introduction: Equity in mathematics education. In P. Rogers & G. Kaiser (Eds.), *Equity in mathematics education: Influences of feminism and culture* (pp. 1–10). London, UK: RoutledgeFalmer.
- Karpov, Y. V. (2003). Development through the lifespan: A neo-Vygotskian approach. In A. Kozulin, B. Gindis, V. S. Ageyev, & S. M. Miller (Eds.), *Vygotsky's educational theory in cultural context* (pp. 138–155). Cambridge, UK: Cambridge University Press. doi:10.1017/cbo9780511840975.009
- Langer-Osuna, J. M. (2011). How Brianna became bossy and Kofi came out smart: Understanding the trajectories of identity and engagement for two group leaders in a project-based mathematics classroom. *Canadian Journal of Science, Mathematics and Technology Education, 11*(3), 207–225. doi:10.1080/14926156.2011.595881
- Langer-Osuna, J. M., & Esmonde, I. (in press). Identity in research on mathematics education. In J. Cai (Ed.), *Compendium for research in mathematics education*. Reston, VA: National Council of Teachers of Mathematics.
- Leontyev, A. N. (2009). *The development of mind: Selected works of Aleksei Nikolaevich Leontyev*. Retrieved from www.marxists.org/archive/leontev/works/development-mind.pdf
- Lewis, G. (2013). Emotion and disaffection with school mathematics. *Research in Mathematics Education, 15*(1), 70–86. doi:10.1080/14794802.2012.756636
- Lim, J. H. (2008). Adolescent girls' construction of moral discourses and appropriation of primary identity in a mathematics classroom. *ZDM, 40*(4), 617–631. doi:10.1007/s11858-008-0119-7
- Lubienski, S., & Ganley, C. (in press). Research on gender and mathematics. In J. Cai (Ed.), *Compendium for research in mathematics education*. Reston, VA: National Council of Teachers of Mathematics.
- Marsh, H. W. (1987). The big-fish-little-pond effect on academic self-concept. *Journal of Educational Psychology, 79*(3), 280–295. doi:10.1037/0022-0663.79.3.280
- Marsh, H. W., Trautwein, U., Lüdtke, O., & Köller, O. (2008). Social comparison and big-fish-little-pond effects on self-concept and other self-belief constructs: Role of generalized and specific others. *Journal of Educational Psychology, 100*(3), 510–524. doi:10.1037/0022-0663.100.3.510

- Martin, D. (2012). Learning mathematics while Black. *The Journal of Educational Foundations*, 26(1–2), 47–66.
- Mehan, H. (1979). *Learning lessons: Social organization in the classroom*. Cambridge, MA: Harvard University Press. doi:10.4159/harvard.9780674420106
- Mendick, H. (2005a). A beautiful myth? The gendering of being/doing “good at maths.” *Gender and Education*, 17(2), 203–219. doi:10.1080/0954025042000301465
- Mendick, H. (2005b). Mathematical stories: Why do more boys than girls choose to study mathematics at AS-level in England? *British Journal of Sociology of Education*, 26(2), 235–251. doi:10.1080/0142569042000294192
- Ministerio de Educación Chile. (2005). *Análisis de las diferencias de logro en el aprendizaje escolar entre hombres y mujeres* [Analysis of achievement differences in school learning between men and women]. Santiago, Chile: SIMCE.
- Mittelberg, D., Rozner, O., & Forgasz, H. (2011). Mathematics and gender stereotypes in one Jewish and one Druze Grade 5 classroom in Israel. *Education Research International*, 2011, 1–10. doi:10.1155/2011/545010
- Mizala, A., Martínez, F., & Martínez, S. (2015). Pre-service elementary school teachers’ expectations about student performance: How their beliefs are affected by their mathematics anxiety and student’s gender. *Teaching and Teacher Education*, 50, 70–78. doi:10.1016/j.tate.2015.04.006
- Mizala, A., & Torche, F. (2012). Bringing the schools back in: The stratification of educational achievement in the Chilean voucher system. *International Journal of Educational Development*, 32(1), 132–144. doi:10.1016/j.ijedudev.2010.09.004
- Nardi, E., & Steward, S. (2003). Is mathematics T.I.R.E.D? A profile of quiet disaffection in the secondary mathematics classroom. *British Educational Research Journal*, 29(3), 345–367. doi:10.1080/01411920301852
- Organisation for Economic Co-operation and Development. (2015). *The ABC of gender equality in education: Aptitude, behaviour, confidence*. Paris, France: OECD Publishing. doi:10.1787/9789264229945-en
- Radovic, D. (2015). *Towards conceptual coherence in research on mathematics learner identity: A systematic review of the literature*. Manuscript in preparation.
- Radovic, D., & Preiss, D. (2010). Patrones de discurso observados en el aula de matemática de segundo ciclo básico en Chile [Patterns of speech observed in the second-cycle basic mathematics classroom in Chile]. *Psykhé (Santiago)*, 19(2), 65–79. doi:10.4067/S0718-22282010000200007
- Ramirez, M. (2007). Diferencias dentro de las salas de clases: Distribución de rendimiento en matemáticas [Differences within classrooms: Distribution of math performance]. *Estudios Públicos*, 106, 5–22.
- Renold, E., & Allan, A. (2006). Bright and beautiful: High achieving girls, ambivalent femininities, and the feminization of success in the primary school. *Discourse: Studies in the Cultural Politics of Education*, 27(4), 457–473. doi:10.1080/01596300600988606
- Rodd, M., & Bartholomew, H. (2006). Invisible and special: Young women’s experiences as undergraduate mathematics students. *Gender and Education*, 18(1), 35–50. doi:10.1080/09540250500195093
- SCM (Version 4.0) [Computer software]. Chapel Hill, NC: University of North Carolina.
- Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, 34(4), 14–22. doi:10.3102/0013189X034004014
- Sinclair, J. H., & Coulthard, R. M. (1975). *Towards an analysis of discourse: The English used by teachers and pupils*. London, UK: Oxford University Press.
- Solomon, Y. (2007). Experiencing mathematics classes: Ability grouping, gender and the selective development of participative identities. *International Journal of Educational Research*, 46(1–2), 8–19. doi:10.1016/j.ijer.2007.07.002
- Solomon, Y. (2008). *Mathematical literacy: Developing identities of inclusion*. London, UK: Routledge.
- Solomon, Y. (2012). Finding a voice? Narrating the female self in mathematics. *Educational Studies in Mathematics*, 80(1), 171–183. doi:10.1007/s10649-012-9384-z

- Solomon, Y., Radovic, D., & Black, L. (2015). "I can actually be very feminine here": Contradiction and hybridity in becoming a female mathematician. *Educational Studies in Mathematics*, *91*(1), 55–71. doi:10.1007/s10649-015-9649-4
- Stentoft, D., & Valero, P. (2009). Identities-in-action: Exploring the fragility of discourse and identity in learning mathematics. *Nordic Studies in Mathematics Education*, *14*(3), 55–77.
- Strauss, A., & Corbin, J. (1998). *Basics of qualitative research* (2nd ed.). Thousand Oaks, CA: Sage.
- Stuven, A. M. (2013). La mujer ayer y hoy: Un recorrido de incorporación social y política [Women yesterday and today: A journey of social and political incorporation]. *Centro de Políticas Públicas UC*, *8*(61), 1–20.
- Varelas, M., Martin, D. B., & Kane, J. M. (2012). Content learning and identity construction: A framework to strengthen African American students' mathematics and science learning in urban elementary schools. *Human Development*, *55*(5–6), 319–339. doi:10.1159/000345324
- Voz de Mujer. (2010). *Primera encuesta nacional sobre mujer y trabajo en Chile* [First national survey on women and labor in Chile]. Santiago, Chile: Comunidad Mujer.
- Voz de Mujer. (2013). *Mujer y trabajo: Factores culturales, roles de género y comportamiento laboral de las chilenas* [Women and work: Cultural factors, gender roles and labor behavior of Chileans]. Santiago, Chile: Comunidad Mujer.
- Walkerdine, V. (1998). *Counting girls out: Girls and mathematics* (2nd ed.). London, UK: RoutledgeFalmer.
- Wells, G. (1993). Reevaluating the IRF sequence: A proposal for the articulation of theories of activity and discourse for the analysis of teaching and learning in the classroom. *Linguistics and Education*, *5*(1), 1–37. doi:10.1016/S0898-5898(05)80001-4
- Wells, G. (1999). *Dialogic inquiry: Towards a sociocultural practice and theory of education*. New York, NY: Cambridge University Press. doi:10.1017/CBO9780511605895
- Williams, J., Davis, P., & Black, L. (2007). Sociocultural and cultural–historical activity. Theory perspectives on subjectivities and learning in schools and other educational contexts. *International Journal of Educational Research*, *46*(1–2), 1–7. doi:10.1016/j.ijer.2007.07.001
- Yin, R. K. (2003). *Case study research: Design and methods*. Thousand Oaks, CA: Sage.
- Ylvisaker, M., McPherson, K., Kayes, N., & Pellett, E. (2008). Metaphoric identity mapping: Facilitating goal setting and engagement in rehabilitation after traumatic brain injury. *Neuropsychological Rehabilitation*, *18*(5–6), 713–741. https://doi.org/10.1080/09602010802201832
- Zeidner, M., & Schleyer, E. J. (1999). The big-fish-little-pond effect for academic self-concept, test anxiety, and school grades in gifted children. *Contemporary Educational Psychology*, *24*(4), 305–329. doi:10.1006/ceps.1998.0985

Authors

Darinka Radovic, Center for Mathematics Modeling, Department of Mathematical Engineering, Facultad de Ciencias Físicas y Matemáticas, Universidad de Chile, Beauchef 851, Santiago, Chile; dradovic@dim.uchile.cl

Laura Black, Institute of Education, The University of Manchester, Oxford Road, Manchester, United Kingdom, M13 9PL; laura.black@manchester.ac.uk

Christian E. Salas, Laboratorio de Neurociencia Cognitiva y Social (LaNCyS), Facultad de Psicología, Universidad Diego Portales, Vergara 275, Santiago, Chile; salasriquelme@gmail.com

Julian Williams, Institute of Education, The University of Manchester, Oxford Road, Manchester, United Kingdom, M13 9PL; julian.williams@manchester.ac.uk

Submitted November 23, 2015

Accepted April 10, 2016