REASONING ABOUT THE RATE OF CHANGE WHILE LINKING CO_2 POLLUTION TO GLOBAL WARMING

PENSANDO EN LA RAZÓN DE CAMBIO AL VINCULAR CONTAMINACIÓN POR CO_2 CON EL CALENTAMIENTO GLOBAL

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This paper characterizes the way three preservice mathematics teachers (PSTs) understand and quantify the rate of change as they model the link between carbon dioxide (CO_2) pollution and global warming. I also discuss what PSTs learned about the concept of forcing by CO_2, a key metric of global warming. The PSTs completed a mathematical task during an individual, task-based interview. The study revealed three levels of understanding of the rate of change in relation to quantitative operations (comparison versus coordination), graphing activity (pointwise versus smooth and continuous), and concavity (discovering versus anticipating). Depending on their level of understanding, PSTs could imagine the rate of change changing discretely or continuously with respect to an independent variable. PSTs also learn four central ideas regarding the forcing by CO_2 as a result of working on the task.

Keywords: Cognition, Modeling, STEM/STEAM, Teacher Education - Preservice

Introduction

Climate change is a pressing issue for this century with potentially irreversible and disastrous consequences for social and natural systems (Intergovernmental Panel on Climate Change [IPCC], 2013). The *United Nations* has called for incorporating climate change education in schools (Anderson, 2012; Global Education Monitoring [GEM], 2016). Since students have different interests and learning abilities, teachers from all disciplines can contribute to climate change education (McKeown & Hopkins, 2010). Mathematics teachers can play a central role in this endeavor since mathematical modeling represents a promising approach for connecting mathematical learning and climate change education (González, 2018, 2019; Barwell & Suurtamm, 2011; Barwell, 2013a, 2013b). Teachers, however, need to be prepared for the challenge, which requires teacher education programs to prepare preservice mathematics teachers (PSTs) for incorporating climate change into their instruction.

Lambert and Bleicher (2013) have identified two key concepts from climate sciences that preservice science teachers need to learn about in order to understand climate change: (a) the Earth's energy balance, and (b) the link between carbon dioxide (CO_2) pollution and global warming. It is reasonable to extend this premise to PSTs since they are less familiar with concepts from climate science than preservice science teachers. Therefore, a starting point may involve studying the energy balance and the link between CO_2 and global warming as dynamic situations where two (or more) variables change together (covariation). In this paper, I characterize, from a covariational reasoning perspective, the way three PSTs think about the rate of change as they model the link between CO_2 pollution and global warming. I also discuss what PSTs learned about the concept of *Forcing by* CO_2, a key metric for assessing the impact of CO_2 pollution on global warming.

Conceptual Framework

Covariational reasoning refers to "the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other" (Carlson,

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Jacobs, Coe, Larsen, & Hsu, 2002, p. 354). Johnson (2015) distinguished two categories of quantitative operations that students use when reasoning about covariation and rate of change: operations of *comparison* (QO-Comp) and operations of *coordination* (QO-Coord). *QO-Comp* involves conceiving a quantity's variation in chunks and produces associations of amounts of change between covarying quantities. The amounts of change in y are compared for (not necessarily equal) amounts of change in x in order to make viable claims about the rate of change. *QO-Coord* involved conceiving a quantity's variation smoothly and produces relationships between covarying quantities. The relationships are coordinated through division to create a new quantity measuring *degrees of change* that supports accurate claims about the rate of change. Carlson and colleagues' concept of covariational reasoning and Johnson's (2015) QO-Comp and QO-Coord informed the discussion about the ways PSTs understood and quantified the rate of change.

Methodology

This paper is part of a larger study that investigated how PSTs make sense of simple mathematical models of climate change. Three secondary PSTs —hereafter Jodi, Pam, and Kris— enrolled in a mathematics education program at a large Southeastern university in the United States participated in that larger study. Here, I focus on their responses to one task of the larger study: the *Forcing by* CO_2 *Task*.

The Forcing by CO_2 Task

The Erath's energy balance accounts for all heat flows (in Joules per second per square meters, or $Js^{-1}m^{-2}$) that there exit in the continuous heat exchange between the sun, the planet's surface, and the atmosphere (Figure 1a). The sun warms up the planet's surface at an approximately constant heat flow S. As the surface heats up, it radiates heat to the atmosphere (R). A small fraction of it escapes to space (L), but the majority (B) is absorbed by *greenhouse gases* (GHG) in the atmosphere. The atmosphere then re-radiates a fraction of the absorbed heat back to the surface (A), further increasing its temperature. The heat flow A represents the magnitude of the *greenhouse effect*, which enhances the planet's mean surface temperature. The energy balance shows that changes in the concentration of GHG result in changes in the planet's mean surface temperature. The Fording by CO_2 Task (Figure 1b) focuses on carbon dioxide (CO_2) because it is a key driver of global warming, as human activity produces large amounts of it by burning fossil fuels (IPCC, 2013).

The task defines the forcing by CO_2 as $F = (S + A) - R$, which is a measure of the warming effect over the planet's surface produced by an instantaneous increase in the atmospheric CO_2 concentration, C, (in parts per million, or ppm). If C increases, then the atmosphere can absorb more heat and, consequently, can radiate more heat towards the surface (A increases). Thus, as C increases, so does F, but $lim_{C\to\infty} F(C) = 45$ since S and R remain constant, which puts a cap on the growth of A and, consequently, on the growth of F. This suggests that F increases asymptotically towards 45 $Js^{-1}m^{-2}$ as C increases, producing an increasing, concave-downward graph.

Data Collection

Each PST completed the task during an 80-minute long, individual, task-based interview (Goldin, 2000). The interview followed a *semi-structured* format and was video recorded and transcribed for analysis. I started the interview by showing each PST a 7-minute long video introducing the concepts of energy balance and greenhouse effect. After the video, the PST and I had a Q&A session in which I summarized the central ideas regarding the energy balance and the greenhouse effect and clarified any questions they may have had about those ideas. The video and Q&A session were meant to provide PSTs with a basic knowledge regarding the energy balance and the greenhouse effect so that they could start working on the task.

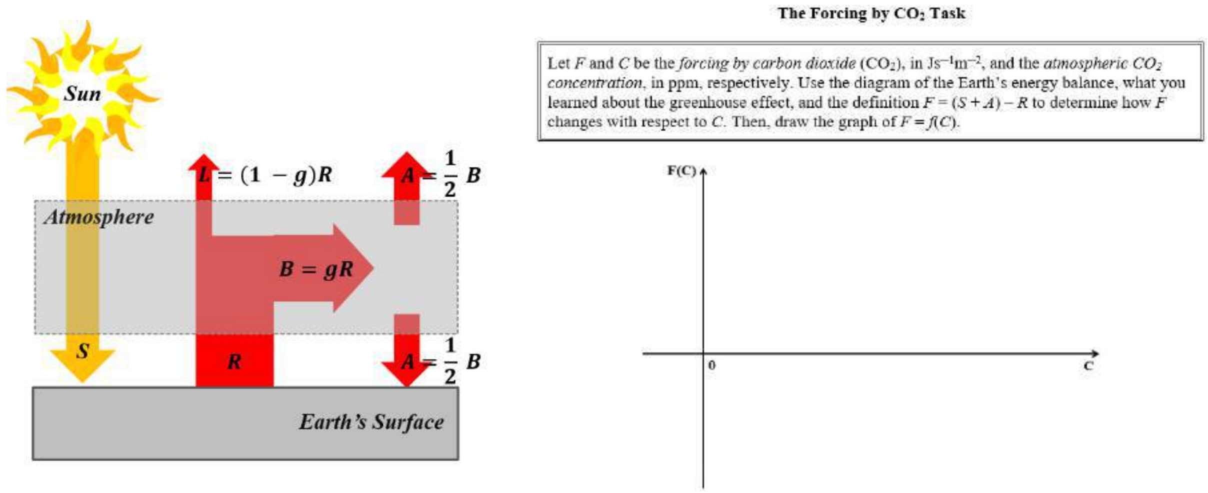

Figure 1. (a) the Earth Energy Balance (Left) and (b) the Forcing by CO_2 Task (Right).

Once the Q&A session ended, PSTs were given the Forcing by CO_2 Task along with a diagram of the energy balance (Figure 1a). The interview had four distinct parts. First, PSTs were asked to think about how F changes as C increases by examining the diagram of the energy balance. The diagram had no values for the heat flows to encourage PSTs to imagine changes happening dynamically. When PSTs experienced difficulties, I gave them initial values for the heat flows so that they could find F-values by using the given definition $F = (S + A) - R$. Second, PSTs had to think about two theoretical scenarios *Scenario 1* described a completely transparent atmosphere (an atmosphere that absorbs no surface heat) and was assumed to happen for $C = 0$ ppm. Scenario 1 corresponded to the minimum forcing (F-value) for the given initial values of the heat flows. *Scenario 2* described a completely opaque atmosphere (an atmosphere that absorbs all surface heat) and was assumed to happen for $C = 1{,}000{,}000$ ppm (highest concentration possible). Scenario 2 corresponded to the maximum forcing (F-value). The PSTs were expected to imagine how F increased from Scenario 1 to Scenario 2 and anticipate the graph's concavity. Third, I introduced the *Excel Simulation*, a spreadsheet that allowed PSTs to enter C-values and obtain the corresponding F-values. The Excel Simulation assisted PSTs in examining and quantifying changes in F for corresponding changes in C and evaluating the accuracy of their graphs. Finally, I asked PSTs to draw the graph of the *Sensitivity of F to C*, or the rate of change of F with respect to C. Here, I examined the PSTs' ability to conceive the rate of change as a measure of sensitivity and as a quantity in and of itself that covaried with C.

Data Analysis

Interview videos and transcripts were analyzed through the *Framework Analysis* (FA) method (Ward, Furber, Tierney, & Swallow, 2013). I watched all videos and divided them into smaller episodes. For each episode, I took notes regarding PSTs' views of forcing, covariational reasoning, and understandings of rate of change. I used the notes to develop an analytic framework, which included six codes about forcing, eight codes regarding covariation, and five codes about rate of change. The analytic framework was applied back to the data to code all episodes. Next, I looked for patterns across the participants' responses and categorized codes into themes. The patterns and themes helped me characterize the way PSTs understand the forcing and the rate of change.

Results

The Direction of Change of the Forcing

During the first part of the interview, PSTs looked at the diagram (Figure 1a) and identified the heat flows that changed when *C* increased and those that remained constant (i.e., unaffected by changes in *C*). In doing so, PSTs thought about how an increment in *C* influences the atmosphere's capacity to absorb and radiate heat (changes in *B* and *A*, respectively), which represents a foundational idea to understand the forcing by CO_2. PSTs inferred the direction in which *F* was changing by utilizing the given definition $F = (S + A) - R$. They noticed that an increase in *C* resulted in an increase in *A*, while the heat flows *S* and *R* remained constant, which meant that *F* increased when *C* increased. For instance, Kris stated "if *B* increases, then *A* is going to increase, and *S* and *R* stay the same [*pauses*]. So, [*F*] is going to be positive".

During the second part of the interview, PSTs thought about *Scenario 1* and *Scenario 2*. They realized that the scenarios represented the minimum and maximum forcing, respectively. For instance, Jodi described Scenario 2 as follows:

A would be 390 over 2, which is going to be [*uses calculator*]. So, A is 195, and we would need, we would want $_S$ to equal A [*writes S = A*]. But, since 240 is greater than 195, we would need to add [*F*] [*writes 240 = 195 + F*]. And, that would make F = 45. In the case we add more CO_2 to the atmosphere and L no longer is emitted

The PSTs assumed Scenario 1 occurred for $C = 0$ and found that $F = (240 + 0) - 390 = -150$ $Js^{-1}m^{-2}$. For Scenario 2, they assumed it occurred for $C = C_M$ and had $F = (240 + 195) - 390 = 45$ $Js^{-1}m^{-2}$. They represented these scenarios in the coordinate plane by the points $(0, -150)$ and $(C_M, 45)$, respectively. Then, Pam and Jodi drew a line incident to both points as the graph of *F* (Figure 2), while Kris could not decide whether the graph should be an increasing, concave-downward curve or an increasing line. She stated that a line "would imply that it is like a constant rate of change with *C* and [*F*]." Kris's understanding of rate of change appeared more advanced than Jodi and Pam's since it involved the realization that the shape of a graph is related to the variation in the rate of change.

The Rate of Change of the Forcing

During the third part of the interview, the *Excel Simulation* was introduced. Here, the PSTs also learned that *F* follows the rule "*F* increases by 4 $Js^{-1}m^{-2}$ every time *C* doubles[1]" which is widely accepted among the experts (Huang & Shahabadi, 2014; IPCC, 2013).

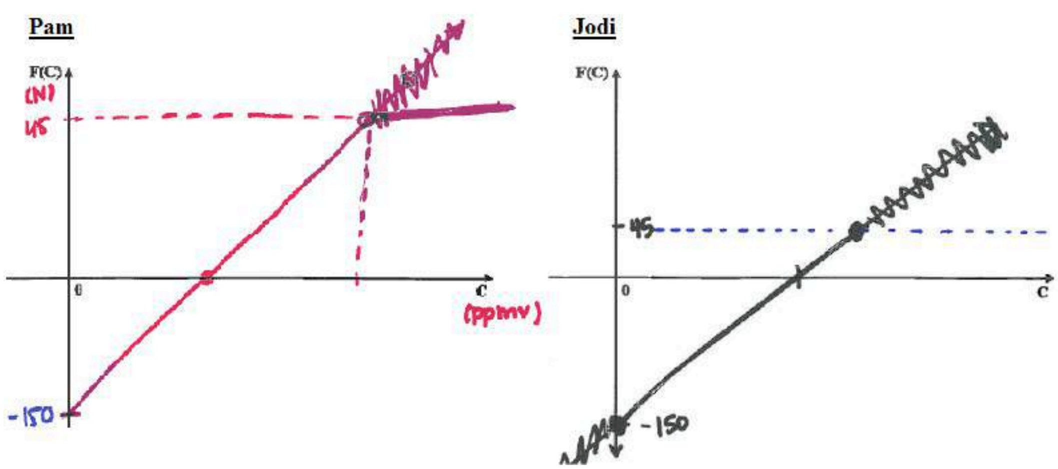

Figure 2. (a) Pam's linear graph of F (left) and (b) Jodi's linear graph of F (right).

[1] A more real estimate is about 3.7 $Js^{-1}m^{-2}$ (IPCC, 2013), but I rounded it to 4 $Js^{-1}m^{-2}$ for simplicity.

PSTs were asked to find out whether *F* was a linear or a nonlinear function of *C*. All three PSTs determined *F*-values corresponding to equally spaced *C*-values. Then, they compared the differences $\Delta_i F$ and noticed they were decreasing, discarding the linear model. After that, PSTs demonstrated three different ways of quantifying the rate of change and understanding its connection to the concavity of the graph of *F*, as they drew new versions of that graph. Pam did not anticipate the concavity of the graph from interpreting the decreasing increments $\Delta_i F$, suggesting she did not see them as an indicator of concavity or a measure of the variation in the *degree of change* of *F* with respect to *C*. Instead, Pam used the rule "*F* increases by 4 $Js^{-1}m^{-2}$ when *C* doubles" to coordinate *C*-values with *F*-values, creating a discrete collection of pairs (*C* , *F*) and drawing the graph of *F* using a pointwise approach (Figure 3a). When finished, Pam said "Oh! This looks like a logarithmic thing I hate", suggesting she did not anticipate the concavity of her graph as much as she *discovered* it.

In contrast, Jodi anticipated the concavity of *F* by interpreting the decreasing increments $\Delta_i F$ as indicating that *F* increased less and less as *C* increased.

So, the relationship is not linear because the change in y over the change in x is not equal between two points. But, I see that, as we increase [C], the change in F is less. So, we may end up getting a function that looks like that *[draws a tiny, increasing, concave-down curve]*

Although Jodi anticipated the concavity, she still used the Excel Simulation to create a discrete collection of pairs (*C* , *F*). She then used a pointwise approach to draw her final version of the graph of *F*. This is an interesting behavior because it suggests that she did not have complete confidence on her interpretation of the differences $\Delta_i F$ in terms of concavity. A possible explanation is that her understanding of those differences as an indicator of concavity and a measure of the degree of change of *F* may have been still stabilizing in her mind.

Finally, Kris anticipated the concavity of *F* by interpreting the decreasing average rate of change of *F*. Her interpretation confirmed her previous suspicion that the graph was an increasing, concave-downward curve.

K: That is really weird, how like, if you look at the change from [*C* = 0] to [*C* = 1] [*pauses*]

I: There is a big jump

K: Yeah, like over a hundred (*F* increases more than 100 $Js^{-1}m^{-2}$). And then you get from [*C* = 10] to [*C* = 20] and it is only like four (*F* increases by approximately 4 $Js^{-1}m^{-2}$). So like, for every change [of] 2.5 [in *C*], [*F*] changes like one-ish. So that is what I was thinking about when I said that [the graph] may look like this [*draws an increasing, concave-downward curve*]

Kris's way of quantifying the average rate of change of *F* supported both anticipating concavity *and* drawing the graph of *F* in a smooth and continuous way (Figure 3b). Also, Kris's use of ratios represents a step forward in the formalization of the concept of rate of change in relation to the comparison of the differences $\Delta_i F$ for equal increments ΔC.

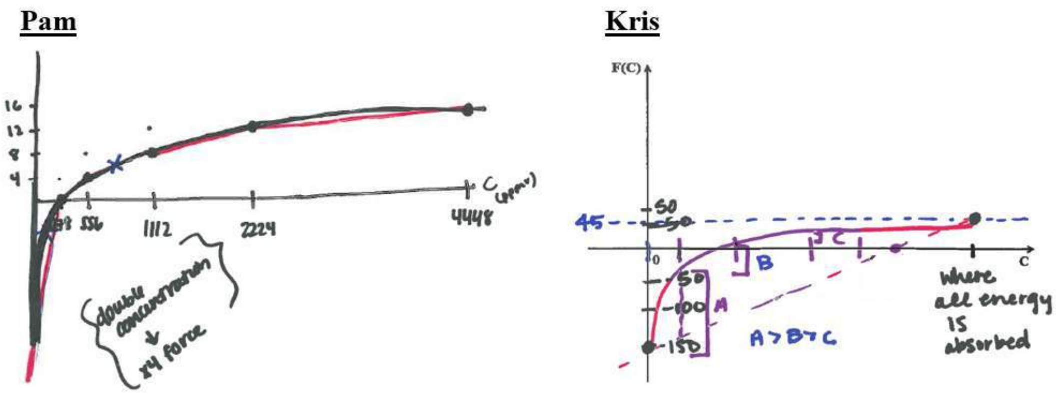

Figure 3. (a) Pam's final graph of F (Left) and (b) Kris's final graph of F (Right).

The Sensitivity of F to C

Pam and Kris participated in the fourth part of the interview involving the *Sensitivity of F to C* (i.e., the rate of change of *F* with respect to *C*). Unfortunately, Jodi did not have time to participate during that last part. The analysis of Pam and Kris's responses suggest two different ways of quantifying the *sensitivity* of *F* to *C* and two different ways of conceiving that sensitivity *covarying* with *C*.

Pam quantified the sensitivity by the *steepness* of the graph of *F* corresponding to unequally-spaced values of *C* (each interval was twice as long as the previous one). She then attended to the variation in the steepness as she moved from one interval of *C* to the next. She translated that variation into degrees of sensitivity (e.g., more or less sensitive).

P: So, it is not super sensitive here [*uses two fingers to indicate the steepness of the graph of F for* $2224 \leq C \leq 4448$]

I: Could you tell me a little bit more about how you figured that out by looking at this graph [*point at her graph of F*]?

P: Here [*points at the interval [0 , 278]*], [*C*] increased a little bit, and the force [*sic*] went crazy [*moves her index finger up quickly to indicate a large increase in F*], I mean compare to everything else, it went higher. Here [*points at the interval [278 , 556]*], [*C*] increased a little bit more, and the sensitivity didn't increase that much. So, [*F*] is not as sensitive when there is more concentration [*moves her fingers to the right to indicate the increase in C*]

The transcript above shows how Pam imagined the steepness decreasing as she moved from one interval of *C* to the next. This helped her identify the direction of change of the *sensitivity* (i.e., it decreases as *C* increases). However, she did not notice that the decline in steepness slowed down as *C* increased, hence she could not anticipate the concavity of the graph of the sensitivity. In order to draw the graph, Pam first found four values of the average rate of change of *F*: 4/278, 4/556, 4/1112, and 4/2224, corresponding to the intervals [278 , 556], [556 , 1112], [1112 , 2224], and [2224 , 4448], respectively. Then, Pam notice that "my concentration increases by double, and my sensitivity goes down by half [*writes 'concentration × 2, sensitivity ÷ 2'*]". She used that rule to create the discrete collection of pairs (278 , 1/2 *F'*(0)), (556 , 1/4 *F'*(0)), (1112 , 1/8 *F'*(0)), and (2224 , 1/16 *F'*(0)), where *F'* represents the sensitivity of *F* to *C*. Then, Pam drew the graph of the sensitivity with a pointwise approach (Figure 4a). This suggests she discovered the concavity of the graph of the sensitivity rather than anticipating it.

In contrast, Kris's ways of quantifying the sensitivity involved thinking in terms of *how resistant F* was to changes in *C*, as indicated by the graph of *F*. When I asked her how the sensitivity changes as *C* increases, Kris replied "sensitivity decreases because [*F*] is more resistant to a change in *C*". She then drew a decreasing, concave-upward graph of the sensitivity in a smooth and continuous way

(Figure 4b). Since she did not justify the concavity of her graph, I asked her to elaborate on how she figured the concavity out, to which she replied:

As we increase $_C$ by equal amounts each time *[uses two fingers to indicate equal increments in C], F is increasing by smaller, and smaller, and smaller amounts [uses two fingers to indicate decreasing increments in F]*. So, it is becoming less sensitive to the changes in $_C$. Because it takes a bigger change in $_C$ to equal the equal change in $_F$.

Because one must double *C* to create the same increment in *F*, she claimed that "the sensitivity decreases at a decreasing rate". Kris's quantification of the sensitivity allowed her to anticipate concavity, draw an accurate graph, *and* make viable claims about the rate of change of the sensitivity. This suggests that Kris not only reasoned about the rate of change of *F*, but also about the rate of change of the rate of change of *F*, which is foundational to understand second derivative in Calculus.

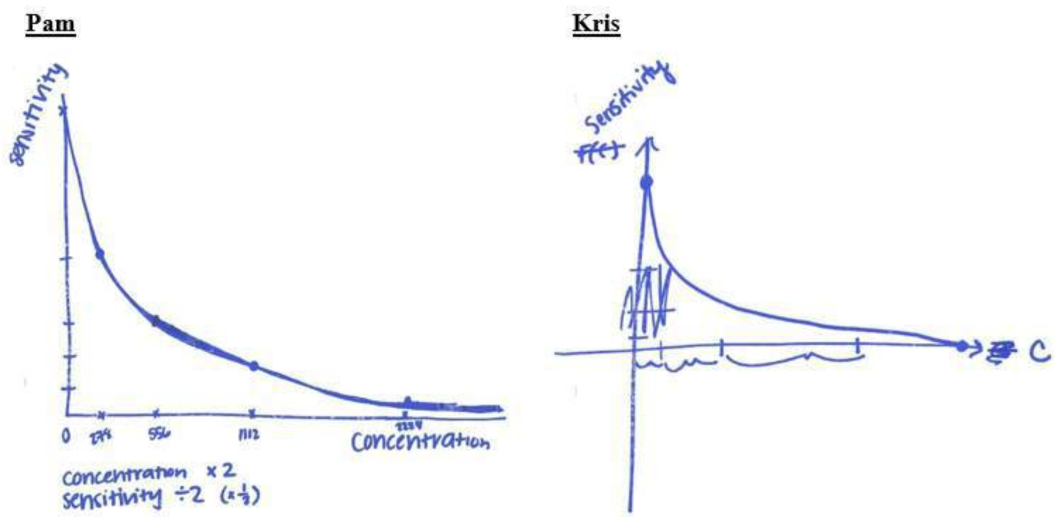

Figure 4. (a) Pam's graph of sensitivity (Left) and (b) Kris's graph of sensitivity (Right).

Finally, by thinking about the sensitivity of *F* to *C*, Pam and Kris learned that the forcing by CO_2 becomes less sensitive to changes in *C* as *C* increases. This is another characteristic of the forcing widely accepted among the experts (Huang & Shahabadi, 2014; IPCC, 2013).

Conclusions

The study revealed three different levels of understanding of the rate of change among the PSTs. *Level 1* is represented by Pam; she did not demonstrate quantitative operations related to reasoning about the rate of change *F*. She created a discrete collection of pairs (*C* , *F*) and used a pointwise approach to draw the graph of *F*. The concavity was discovered after finishing the graph and no viable claims about the rate of change were made. *Level 2* is represented by Jodi; she associated changes $\Delta_i F$ with equal changes ΔC and compared those associations (QO-Comp) to anticipate concavity and make viable claims about the rate of change. She, however, created a discrete collection of pairs (*C* , *F*) and used a pointwise approach to draw the graph of *F*. This suggests that her understanding of the relationship between a graph's shape and the rate of change was not completely stable in her mind. *Level 3* is represented by Kris; she coordinated changes $\Delta_i F$ with changes ΔC through division (QO-Coord) to create a single quantity that allowed her to anticipate concavity, make viable claims about the rate of change, and draw an accurate graph of *F*.

The analysis of Pam and Kris's responses suggest two different ways of quantifying the *sensitivity* of *F* to *C* and two different ways of conceiving covariation between the sensitivity and *C*. Pam quantified the sensitivity by the *steepness* of the graph of *F* corresponding to an interval of *C*. This allowed her to identify the direction of change of the sensitivity (i.e., it decreases when *C* increases). Then, she *compared* (QO-Comp) the values of the average rate of change of *F* for consecutive, unequally-long intervals of *C* (each interval was twice as long as the previous one) in order to define a correspondence rule between values of sensitivity and values of *C*: the sensitivity halves every time *C* doubles. Pam's QO-Comp allowed her to draw an accurate graph but did not support the ability to make claims about the rate of change of the sensitivity or anticipate concavity. In contrast, Kris quantified the sensitivity as the *resistance* of *F* to changes in *C*, as defined by the graph of *F*. This allowed her to identify the direction of change of the sensitivity (i.e., it decreases when *C* increases). Then, she *coordinated* (QO-Coord) changes in resistance with changes in *C* in order to draw an accurate graph of the sensitivity in a smooth and continuous way, make claims about the rate of change of the sensitivity, and anticipate concavity. Most interestingly, Kris's QO-Coord supported reasoning about the rate of change of the rate of change of *F*, a key idea to understand the second derivative in Calculus (Johnson, 2012).

Finally, the study also shows that PSTs learned four important aspects about the *forcing by* CO_2: (1) an increase in atmospheric CO_2 concentration enhances the atmosphere's capacity to absorb and radiate heat, which further warms the planet's surface; (2) the forcing has a theoretical minimum value, when the atmosphere absorbs no surface heat, and a theoretical maximum value, when the atmosphere absorbs all surface heat; (3) the doubling CO_2 rule for the forcing (*F* increases by 4 $Js^{-1}m^{-2}$ every time *C* doubles); and (4) the forcing by CO_2 becomes less sensitive to changes in *C* as *C* increases.

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