Quantitative Reasoning and Thinking About Systems: The Case of Climate Change

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This paper discusses the role of quantitative reasoning in developing an understanding of the Erath's energy budget, a key concept in climate science, as a system formed by multiple interrelated energy flows between the sun, the surface, and the atmosphere. The paper illustrates these claims by comparing the quantitative reasoning of two preservice mathematics teachers (PSTs). First, the PSTs must understand the quantities (parts) involved in the energy budget: concentration, irradiance, and their units of measure. Second, they must make sense of the interrelationships between those quantities in the context of the energy budget. It is concluded that a sophisticated quantitative understanding of the energy budget as a system can support the understanding of climate change.

Introduction

Human activities (e.g., electricity generation, transportation, or food production) release large amounts of greenhouse gases into the atmosphere, which trap heat and enhance the average temperature of the planet. The Intergovernmental Panel on Climate Change (IPCC) has warmed us against exceeding a global warming of 1.5 ºC above the preindustrial era average, otherwise we will witness devastating and irreversible consequences to our social, economic, and natural systems. We have, at most, 30 years before passing that threshold (IPCC, 2018), and staying within that safe limit requires everyone's commitment to support and adopt mitigation strategies, which is more likely when people possess knowledge about climate change (Sewell et al., 2017).

Unfortunately, climate change is not a phenomenon easy to understand; its planetary scale makes it difficult for a single person to experience all of its consequences, and its complexity requires the person to deal with concepts from systems dynamics such as conceiving multiple interrelated variables (interconnectedness), identifying causality loops between variables (feedback), and examining patterns of variation over time (dynamic relationships) (Ghosh, 2017; Roychoudhury et al., 2017; Schuler et al., 2018).

Mathematics and mathematics education can play a prominent role in helping students and teachers grasp those concepts, thus promoting climate change education (Barwell, 2013a, 2013b; González, 2021; Renert, 2011). A promising approach involves examining, from a quantitative perspective, a key concept in the study and modeling of climate change: The *Earth's energy budget* (Lambert & Bleicher, 2013). This paper offers a theoretical discussion about the role of quantitative reasoning in developing an understanding of the energy budget as a system formed by multiple interrelated energy flows (quantities) between the sun, the surface, and the atmosphere. The discussion uses examples from my previous research on how preservice mathematics teachers (PSTs) make sense of the mathematics involved in modeling climate change (González, 2017).

The Earth's Energy Budget

The Earth's energy budget accounts for the direction and magnitude of all energy flows that exist between the sun, the planet's surface, and the atmosphere (Figure 1). First, the sun radiates energy and heat towards the Earth at an approximately constant rate; most of it passes through the atmosphere warming the planet's surface (*S*). As the surface heats up, it radiates infrared energy upward, to the atmosphere (*R*). A small fraction of it escapes to space (*L*), but the majority (*B*) is absorbed by *greenhouse gases* (GHG) warming the atmosphere. As the atmosphere heats up, it radiates a fraction of the absorbed energy in both directions: out to space and back to the surface (*A*'s). The latter further increases the temperature of the surface and represents the magnitude of the *greenhouse effect*. This energy exchange occurs continuously and dynamically between the sun, the surface, and the atmosphere, and regulates the planet's average temperature.

The energy flows *S*, *R*, *L*, *B*, *A* (Figure 1) can change in magnitude due to different factors known as climate forcings. A particularly interesting forcing in modeling climate change involves determining changes in the energy flows due to changes in the abundance of GHG. The energy flows are quantified in terms of *irradiance* —the energy incident to a surface per unit time per unit area—, which is measured in Joules per second per square meter ($Js^{-1}m^{-2}$) and the abundance of GHG in the atmosphere is quantified in terms of *concentration* — the relative abundance of a gas in the air (mixture of gases) with respect to the air volume—, which is measured in parts per million (ppm) or parts per billion (ppb).

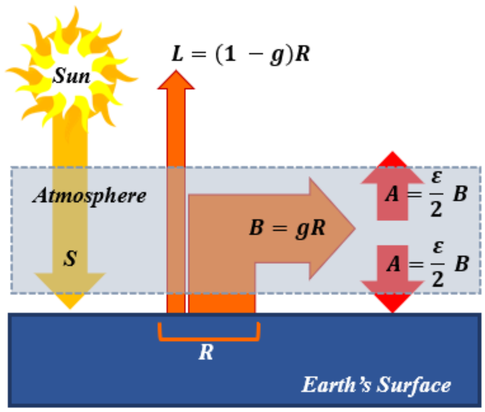


Figure 1. Diagram of the Earth's energy budget.

Theoretical Framework

Current global issues such as sustainability, pollution, climate change, and poverty require systems thinking, or the ability to: (i) visualize the interconnections and relationships between the parts in the system; (ii) examine behavior that changes over time; and (iii) examine how systems-level phenomena emerge from interactions between the system's parts (Orgill, York, & MacKellar, 2019). From a mathematical perspective, understanding those issues systemically involves the construction of quantities associated with the system components and the definition of relationships between the quantities to connect such components. This is when systems thinking intersects with quantitative reasoning since it represents the set of "mental actions of a student who conceives of a mathematical situation, constructs quantities in that situation, and then relates, manipulates, and uses those quantities to make a problem situation coherent" (Weber, Ellis, Kulow, & Ozgur, 2014, p. 24).

The theory of *quantitative reasoning* (Thompson, 2011) is based on the argument that students construct a quantity through an effortful cognitive process known as *quantification*. This process involves identifying measurable attributes of objects and anticipating ways of measuring them. For instance, the quantity concentration of CO_2[1] is involved in situations where several

[1] Carbon dioxide (CO_2) is a main driver of global warming (IPCC, 2018).

gases are mixed together (object). In this mixture, some gases are more abundant than others since they represent a larger fraction of the mixture (measurable attribute). Determining the fraction that correspond to CO_2 is a way of measuring the relative abundance of that gas and such fraction can be measure in units such as parts per million (ppm). According to Thompson, the meaning a student construct for a quantity is inseparable from the quantification process.

In the context of this paper, understanding the energy budget thus requires developing meaning for the quantities representing the abundance of GHG in the atmosphere and the intensity of the energy flows between the sun, the surface, and the atmosphere. It also requires determining the relationships that exit between those quantities that make the energy budget work as a whole.

Context of the Study

This paper came from a larger study that investigated how preservice mathematics teachers (PSTs) understand the mathematics behind simple mathematical models for climate change (González, 2017). The larger study had three phases and included six mathematical tasks. Phase 1 was an examination of the PSTs' conceptions of quantities that commonly appear in mathematical description of climate change: *gas concentration*, in parts per million (ppm), and *irradiance*, in Joules per second per square meter ($J\ s^{-1}m^{-2}$). Phase 2 investigated the ways PSTs reasoned covariationally while making sense of the link between carbon dioxide (CO_2) pollution and global warming. Phase 3 assessed the PSTs' ability to understand more sophisticated models of climate change. Three female PSTs (hereafter Kris, Pam, and Jodi) enrolled in a mathematics education program at a large public university in the Southeast of the United States participated during the larger study. These PSTs have completed two calculus courses, an introduction to higher mathematics course, and a mathematics content course for secondary teachers. The PSTs completed each task during an individual, task-based interview (Goldin, 2000) that lasted about 60 minutes. The interviews were video recorded and transcribed for the analysis.

In what follows, I contrast the quantitative reasoning of Kris and Pam in relation to their work on the first three tasks. These two PSTs were selected because their quantitative reasoning showed interesting differences, and this paper focuses on the first three tasks because those examined the quantities associated with the energy budget's components and relationships between such quantities representing processes in the energy budget.

Results

Before PSTs could understand the energy budget's components, it was necessary to examine their conceptions of the quantities associated with such components: *concentration* and *irradiance*. It was important to explore their conceptions because they may have an impact on their understanding of the different components of the energy budget to which these quantities are associated. In particular, the PSTs should construct concentration as a measure of abundance of a gas in a mixture and irradiance as a measure of intensity of radiation over a surface.

The Diving Tank Task 2 (Table 1) required PSTs to compare the air[2] of two diving tanks (objects) in relation to their content of CO_2. Both, Kris and Pam, made use of concentration in ppm, which they conceptualized as a *volume of CO_2 per 1,000,000 cm^3 of air* (measurable attribute), hence 1 *ppm* represented 1 cm^3 of CO_2 per every 1,000,000 cm^3 of air. For example, Pam described a concentration of 362 ppm as follows: "when I look at that [*points at 362 ppm*], I think automatically [*writes '362/1,000,000'*]". Kris interpreted the same value as follows: "The

[2] Air is a mixture of several gases, one of which is carbon dioxide (CO_2).

ppm of tank A is 362, and that represents the volume of a gas, which is just CO_2, contained in 1,000,000 cm^3 of air".

Table 1. The tasks involving concentration, in ppm, and irradiance, in $Js^{-1}m^{-2}$

Diving Tank Task 2

The *volume concentration* of gas *X*, denoted as Q_x, in an air mixture is the ratio:

$$Q_x = \frac{\text{volume of gas X}}{\text{volume of air}}$$

Diving tanks also contain a small volume of carbon dioxide (CO_2). The table below shows the volumes of air and CO_2 of two diving tanks.

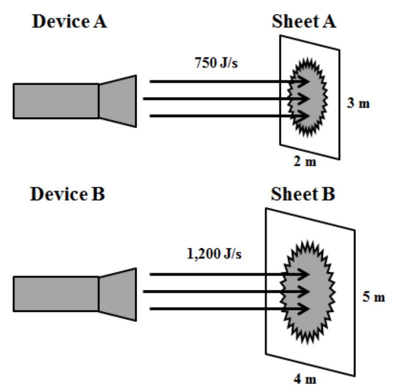
Tank	Air (cm^3)	CO_2 (cm^3)
A	4,000,000	1,448
B	800,000	316

a) Calculate each tank's volume concentration of CO_2. Interpret your result in the context of this situation.

b) When concentrations are small, they are often measured in ppm (parts per million), or the number of parts corresponding to a particular gas in 1,000,000 parts of air. Calculate each tank's *ppm concentration* of CO_2.

Radiation Task 2

When the *energy density* of a metallic sheet of *daridium* increases by 2,500 J/m^2, the sheet's temperature rises by 4 °C. In an experiment, two sheets were positioned at the same distance from two devices that produce radiation (see Figure).



Device A radiates 750 J/s (Joules per second) toward sheet A and device B radiates 1,200 J/s toward sheet B. If both sheets were at room temperature (around 15 °C) at the beginning of the experiment and both devices started radiating energy at the same time, then which sheet will first reach a temperature of 25 °C?

The Radiation Task 2 (Table 1) required PSTs to compare the intensity of radiation received by two very thin metallic sheets (objects). They, after some assistance, made use of irradiance to do the comparison, but they demonstrated different conceptions of that quantity. Kris, for instance, conceptualized irradiance as an *amount by which the energy per m^2 increases per second* (measurable attribute), hence 1 $Js^{-1}m^{-2}$ represented 1 (Jm^{-2}) increase in energy per m^2 per every second. She explained the situation as follows.

> for every second device A is running, 750 … Joules of radiation get put into sheet A. … so we divided 750 by 6, which is the area of sheet A, … we got 125 Joules per meter square [*sic.*] increase in energy density per second that device A is running.

Kris also identified a second measurable attribute for irradiance: *how fast the sheet's temperature increases* ("As long as we know the increase of energy density per second, then we can tell immediately … which [sheet] is going to reach [25 °C] faster"). In contrast, Pam conceptualized irradiance as an association between *an amount of energy and 1 m^2 and 1 s* (measurable attribute), hence 1 $Js^{-1}m^{-2}$ represented 1 J of energy per every (m^2 s) ("It's like every second, how many Joules are to one meter [*sic.*]. So, after one second it's been [*draws a rectangle formed by six squares and writes '125' inside each square*]"). Pam, unlike Kris, did not see a relationship between irradiance and temperature, explaining that irradiance was insufficient information to determine how fast each sheet's temperature was rising ("I think this [*circles 125 $Js^{-1}m^{-2}$*] doesn't help you [complete the task] unless you know how big [the sheet] is or you know the rate").

Then, the PSTs worked with the previous quantities and defined relationships between them in the context of the energy budget (Figure 1). González (2017) asked Kris and Pam to define the planetary energy imbalance, N, a quantity that indicates the magnitude of an energy imbalance in the energy budget. Kris conceptualized N as a difference between the total energy inflow and the total energy outflow at the surface level, $N = (S + A) - R$, so that it expresses the *net change* in surface energy and surface temperature (measurable attributes). For instance, Kris interpreted $N > 0$ as follows: "temperature is increasing because as we gain energy, as we say at the beginning, the temperature increases". Similarly, Kris interpreted $N < 0$ as follows: "temperature is decreasing when N is less than zero [*writes '$T\downarrow$ when $N < 0$'*], because this [*places hand over the diagram of the energy budget*] is losing energy". Pam also conceptualized N as a difference, but hers was a difference between the total amount of energy absorbed and the total amount of energy released by the surface. She explicitly referred to the energy flows as "amounts of energy" and even compared N to an amount of water, stating that N can never be negative ("So, if I put in a cup of water [*points at S and A*], you can't take a cup and a half out [*points at R*], and then have a negative amount of water [*points at the surface*]"). This suggested that Pam conceptualized N as the *actual surface energy* (measurable attribute). Claiming that $S + A < R$ was impossible has a couple of implications. First, Pam seemed to think that the initial surface energy was zero. Second, and related to the first implication, Pam seemed to think that $N = 0$ is equivalent to having zero surface energy. It follows that the case $S + A < R$ is impossible because the surface cannot contain a negative amount of energy.

Next, Kris and Pam examined other relationships while working on the Forcing by CO2 task (González, 2017), which examines how the energy flows change when the atmospheric CO_2 concentration, C, changes (Figure 2). If C were to increase, then so would the atmosphere's capacity to absorb energy. Thus, the atmospheric energy flows B and A would increase, while the energy flow L would decrease. This causes an energy imbalance or *forcing by CO_2* with magnitude $F = (S + A) - R$[3]. By examining the diagram of the energy budget (Figure 1), Kris conceptualized the dependence of B, A, and L on C as well as the independence of S and R on C. She also conceptualized the relationship between B and A, and between F and S, R, and A.

> If you increase the concentration of CO_2, B is going to increase because there are more CO_2 molecules to absorb the energy, so less it is going to be leaked [*points at L*] … OK, so S stays the same [*pauses*]. Wait, hold on [*writes '$A = B/2$'*]. If B increases, then A is going to increase, and S and R stay the same [*pauses*]. So, [F] is going to be positive.

Pam needed additional assistance from the researcher and access to particular values to establish the aforementioned relationships. First, I gave her the values $S = 240$, $R = 390$, L = 90, $B = 300$, and A = 150, all in J $s^{-1}m^{-2}$, in order to illustrate a balance of energy (radiative equilibrium) since $F = (240 + 150) - 390 = 0$. Then, I changed the value $B = 300$ to $B = 340$ in order to simulate the effects of increasing C, and she and I discussed that effect over the energy flows.

I: Let's imagine we increase the concentration of CO_2 by a certain amount. This results in B growing from 300 to 340. So, this flow changed [*point at B*], this flow changed [*point at L*], and these two changed [*point at both A's*] … [S] is still 240 because we are just making changes in the atmosphere, and S does not depend on the atmosphere's composition. So, S is 240 and R remains at 390 as well

P: Except B [*writes "$F = (S + A) - R$"*] … So, B is 340; that means A is now 170. So, we have 240 plus 170 minus 390 [*writes "$F = (240 + 170) - 390 = 20$"*]

I: This value [*point at 20*] is a change the energy budget caused by a change in the concentration of CO_2. That is a forcing by CO_2, that is the value of F

P: Ah! So, when the CO_2 increases, F increases

[3] In the Forcing by CO_2 task, the forcing, F, represents the initial magnitude of N **<u>caused only</u>** by an instantaneous pulse of CO_2 to the atmosphere at $t = 0$. Thus, it is possible to use $F = (S + A) - R$ to find the value of the forcing.

The discussion appeared to have helped Pam understand the relationship between *B*, *A*, on *C*, and the relationship between *F* and *S*, *R*, and *A*, which she synthesized into the relationship *F* = *f*(*C*).

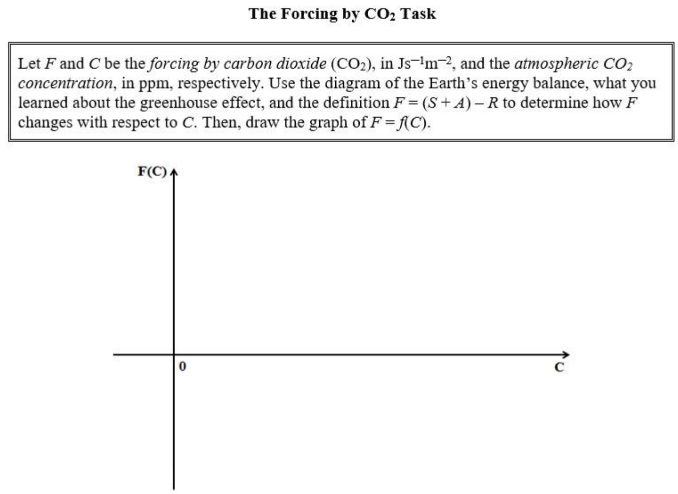


Figure 2. The forcing by CO_2 task

Defining the relationships between the different quantities involved in the energy budget allowed the PSTs to understand the overall the effect of the increase in *C* over the surface temperature. For instance, Pam interpreted her graph of *F* = *f*(*C*) as follows.

> When the forcing by CO_2 is positive, so that means there is more [energy] going in than coming out. Because your concentration is bigger, your *B* is bigger and your *L* is smaller. Because, as we go up [*with her index finger, follows her graph's shape*], the *B* gets bigger, and bigger, and bigger, and the *L* gets smaller, and smaller, and smaller … So, there is more [energy] going into the Earth, so it is hotter, the temperature of the Earth is hotter.

The excerpt illustrates that the PSTs could develop an understanding of the energy budget as a system that responds in a particular way (with an increase in surface energy and surface temperature) when the atmospheric CO_2 concentration increases. That is a key realization to understand the link between CO_2 pollution and global warming (Lambert & Bleicher, 2013).

Discussion

Quantitative reasoning mediated Kris and Pam's ability to understand the energy budget as a system by representing its components and processes through quantities and relationships between them. Kris conceptualized the energy flows between the sun, the surface, and the atmosphere as rates measuring *how fast* the temperature of each component increases. She then defined the energy imbalance, *N*, with a difference between rates so that it indicates the net change in surface energy and surface temperature. Pam conceptualized the energy flows as total amounts of energy transferred between two components of the energy budget. Thus, when she defined *N* with a difference between energy flows, she understood *N* as the actual magnitude of surface energy. Kris's conceptualization of the energy flows appeared to support the realization that changes in the energy budget produce changes in surface temperature. Pam's conceptualization, in contrast, did not support such realization, which may represent an obstacle to understand the role of the energy budget in regulating the Earth's surface temperature.

Kris and Pam's conception of concentration measured in ppm appeared sufficient to conceive the quantity atmospheric CO_2 concentration, *C*, and visualize changes in its magnitude. Then, the PSTs defined relationships between that quantity *C* and the energy flows to construct the quantity forcing by CO_2 as a function of *C*, or *F* = *f*(*C*). Defining that relationship supported the

PSTs' ability to: (i) understand how the energy budget, as a system, responds to an increase in atmospheric CO_2, and (ii) connect changes in CO_2 concentration with changes in surface temperature. These two realizations are key to link CO_2 pollution to global warming and thus develop an understanding of climate change as a human driven phenomenon. These results indicate that mathematics education can play an important role in developing awareness about climate change, addressing skepticism, and promoting support for mitigation policies.

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